THE DERIVATION OF THE RIEMANN ZETA FUNCTION FROM EULER'S QUADRATIC EQUATION AND THE PROOF OF THE RIEMANN HYPOTHESIS

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ABSTRACT
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In this work, we seek to present the connections between the equations constructed from Euler's quadratic equation by Enoch (2015) that gave its Eigen values as the zeros of the Riemann Zeta equation. It was from this quadratic equation that Euler obtained the first few prime numbers for $1 \leq x \leq 40$.A transformation of Enoch (2015) is presented and the same equation obtained byEuler (1749) and Riemann (1859) on the zeta function is gotten. Through this, we now have an equation from Enoch that gives the Riemann zeta equation.

Keywords:*Riemannzeta function; Euler's equation; Meromorphic Function; Non-trivial zeros.*

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Introduction

Euler discovered that (1) is prime for $x = 0. . . 39$.

$$x^2 + x + 41 \tag{1}$$

It is also a known fact that (2) is also a prime for $x = 1. . .39$.

$$x^2 - x + 41 \tag{2}$$

The roots of (1,2) will be $x = -0.5 \pm 6.3836i$ and $x = 0.5 \pm 6.3836i$respectively.

Looking at the structure of these equations, one proposes the equation of the form:

$$\left(kz^2 - kz + p(t)\right)(z + 2n) \tag{3}$$

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It is interesting to know that the roots of this polynomial are just the same as the trivial and the non-trivial zeros of the Riemann Zeta function at some values of $p(t)$.

The Author in his works [15,16] has shown that the Meromorphic functions that are equivalent to the Riemann zeta function are given as:

$$\zeta_E(z) = z(z-1)\left(k + \frac{p(t)}{z(z-1)}\right)(z+2n) \qquad (4)$$

Or

$$\zeta_E(z) = \frac{(kz^2 - k\,z + p(t))(z+2n)}{(e^{z-1}-1)} \qquad (5)$$

Wherein he transformed (4) and (5) into matrices whose Eign values are the trivial and nontrivial spectral points of the Riemann zeta function provided that;

$$p(n) = 800.162 + 968.548n(j) \qquad (6)$$

Or

$$p(n) = 800.162 + 968.548n^{v(j)} \qquad (7)$$

Or

$$P(n) = 1 + kt^2 \ \ where\ k = 4 \qquad (8)$$

The Transformation of (4) into the Riemann Zeta Function

Let

$$\frac{\zeta_E(z)}{(z-1)} = z(z-1)\left(k + \frac{p(t)}{z(z-1)}\right)(2n)\left(1 + \frac{z}{2n}\right) \qquad (9)$$

From [1], Riemann gave

$$-\prod(z/2) \text{ as} \sum_{n\geq 1}^{\infty}\left(1 + \frac{z}{2n}\right) \text{ which is the same as } -\frac{z}{2}\Gamma(z/2)$$

Taking $-\prod(z/2) = -\frac{z}{2}\Gamma(z/2) = \sum_{n\geq 1}^{\infty}\left(\frac{2n+z}{2n}\right)$, we can write (9)as;

Such that

By the process of discretization (11) becomes:

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$$\partial_n(z) = \sum_{n\geq 1}^{\infty} z(z-1)\left(k + \frac{p(t)}{z(z-1)}\right)(2n)\left(1 + \frac{z}{2n}\right) \quad (12)$$

$$\Rightarrow z(z-1)\left(k + \frac{p(t)}{z(z-1)}\right)\sum_{n\geq 1}^{\infty}(2n)\left(1 + \frac{z}{2n}\right) \quad (13)$$

$$\Rightarrow -z^2(z-1)n\Gamma(^z/_2)\left(k + \frac{p(t)}{z(z-1)}\right) \quad (14)$$

From

$$-\frac{\zeta(z)}{z} = \int_0^{\infty} frac\left(\frac{1}{t}\right)t^{z-1}dt \quad (22a)$$

$$-1 = \frac{z}{\zeta(z)}\int_0^{\infty} frac\left(\frac{1}{t}\right)t^{z-1}dt$$

$$\partial_z(z) = \sum_{n=1}^{\infty}\frac{\zeta_n(z)}{(z-1)} = z^2(z-1)n\Gamma(^z/_2)\left(k + \frac{p(t)}{z(z-1)}\right)\left(\frac{z}{\zeta(z)}\int_0^{\infty} frac\left(\frac{1}{t}\right)t^{z-1}dt\right) \quad (14)$$

$$\partial_z(z) = \sum_{n=1}^{\infty}\zeta_n(z) = z^2(z-1)^2n\Gamma(^z/_2)\left(k + \frac{p(t)}{z(z-1)}\right)\left(\frac{z}{\zeta(z)}\int_0^{\infty} frac\left(\frac{1}{t}\right)t^{z-1}dt\right) \quad (14)$$

$$\zeta(z) = \frac{z^3}{\sum_{n=1}^{\infty}\zeta_n(z)}(z-1)^2n\Gamma(^z/_2)\left(k + \frac{p(t)}{z(z-1)}\right)\left(\int_0^{\infty} frac\left(\frac{1}{t}\right)t^{z-1}dt\right) \quad (14)$$

Recall from Euler;

$$\zeta(z) = \prod_p\left(1 - \frac{1}{p^z}\right)^{-1} \quad (15)$$

It can be shown that

$$\zeta(z) = \sum_{n\geq 1}\frac{1}{n^z} = \prod_p\left(1 - \frac{1}{p^z}\right)^{-1}; \quad (16)$$

It is interesting to know that the roots of this polynomial are just the same as the trivial and the non-trivial zeros of the Riemann Zeta function at some values of $p(t)$.

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$$\partial_E(z) = \sum_{n=1}^{\infty}\frac{\zeta_n(z)}{(z-1)} = z^2(z-1)n\Gamma\left({}^{z}/_{2}\right)\left(k+\frac{p(t)}{z(z-1)}\right)\left(\frac{z}{\zeta(z)}\int_0^{\infty} frac\left(\frac{1}{t}\right)t^{z-1}dt\right) \quad (14)$$

$$\partial_E(z) = \sum_{n=1}^{\infty}\zeta_n(z) = z^2(z-1)^2 n\Gamma\left({}^{z}/_{2}\right)\left(k+\frac{p(t)}{z(z-1)}\right)\left(\frac{z}{\zeta(z)}\int_0^{\infty} frac\left(\frac{1}{t}\right)t^{z-1}dt\right) \quad (14)$$

$$\zeta(z) = \frac{z^3}{\sum_{n=1}^{\infty}\zeta_n(z)}(z-1)^2 n\Gamma\left({}^{z}/_{2}\right)\left(k+\frac{p(t)}{z(z-1)}\right)\left(\int_0^{\infty} frac\left(\frac{1}{t}\right)t^{z-1}dt\right) \quad (14)$$

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$$= \sum_{n\geq 1} \prod_{i=1}^{k_n} \frac{1}{P_i^{\propto_i z}} \tag{17}$$

$$= \sum_{n\geq 1} \frac{1}{\prod_{i=1}^{k_n} \frac{1}{P_i^{\propto_i z}}} \Rightarrow n^z = \prod_{i=1}^{k} P_i^{\propto_i z} \tag{18}$$

$$= \prod_{p} \sum_{k\geq 0} \frac{1}{p^{kz}} = \prod_{p} \left(\frac{1}{1 - {}^{1}/_{p^z}} \right) \tag{19}$$

From the above $(18) n^z = \prod_{i=1}^{k} P_i^{\propto_i z}$ implies

$$n = \prod_{i=1}^{k} P_i^{\propto_i} \tag{20}$$

$$\text{Thus from } (14) \quad 2n = 2 \prod_{i=1}^{k} P_i^{\propto_i} \tag{21}$$

$$2 = \sum_{n=1}^{\infty} \frac{1}{2^{n-1}} \qquad \textit{such that} \qquad 2n = \sum_{n=1}^{\infty} \frac{n}{2^{n-1}} \tag{21a}$$

$$2n = \sum_{n=1}^{\infty} \frac{1}{2^{n-1}} \prod_{i=1}^{k} P_i^{\propto_i} \tag{21b}$$

It follows from equations (14) and (21) that:

$$\partial_E(z) = \frac{z}{2}(z-1)\Gamma\left({}^{z}/_{2}\right)\left[-\frac{2}{(z-1)} \prod_{i=1}^{k} P_i^{\propto_i} \left\{ \left(zk + \frac{p(t)}{(z-1)} \right) \right\} \right] \tag{22}$$

It also follows from equations (14) and (21b) that:

$$\partial_E(z) = \frac{z}{2}(z-1)\Gamma\left({}^{z}/_{2}\right)\left[-\frac{1}{(z-1)} \sum_{n=1}^{\infty} \frac{1}{2^{n-1}} \prod_{i=1}^{k} P_i^{\propto_i} \left\{ \left(zk + \frac{p(t)}{(z-1)} \right) \right\} \right] \tag{22a}$$

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$$\partial_E(z) = \frac{z}{2}(z-1)\Gamma\left({}^{z}/_{2}\right)\left[\frac{z}{\zeta(z)}\int_0^\infty frac\left(\frac{1}{t}\right)t^{z-1}dt\left[\frac{2}{(z-1)}\prod_{i=1}^{k}P_i^{\alpha_i}\left\{\left(zk+\frac{p(t)}{(z-1)}\right)\right\}\right]\right] \quad (22)$$

Or we can write equations (22) and (22a) as;

$$\partial_E(z) = \frac{\Gamma\left({}^{z}/_{2}\right)}{z(z-1)}\left[-z^2(z-1)\prod_{i=1}^{k}P_i^{\alpha_i}\left\{\left(zk+\frac{p(t)}{(z-1)}\right)\right\}\right] \quad (22b)$$

and

$$\partial_E(z) = \frac{\Gamma\left({}^{z}/_{2}\right)}{z(z-1)}\left[-\frac{z^2}{2}(z-1)\sum_{n=1}^{\infty}\frac{1}{2^{n-1}}\prod_{i=1}^{k}P_i^{\alpha_i}\left\{\left(zk+\frac{p(t)}{(z-1)}\right)\right\}\right] \quad (22c)$$

Recall from Riemann (1859);

$$\varepsilon(z) = \prod\left({}^{z}/_{2}\right)(z-1)\pi^{-z/2}\zeta(z) \quad (23)$$

Equation (23) Implies that:

$$\varepsilon(z) = \frac{z}{2}\Gamma\left({}^{z}/_{2}\right)(z-1)\pi^{-z/2}\zeta(z) \quad (24)$$

It can be seen that:

$$\zeta(z) = \frac{2\varepsilon(z)\pi^{z/2}}{z(z-1)\Gamma\left({}^{z}/_{2}\right)} \quad (25)$$

From (24) and (25), it can be shown that

$$\varepsilon(z) = \frac{z}{2}\Gamma\left({}^{z}/_{2}\right)(z-1)\pi^{-z/2}\left[\prod_p\left(1-\frac{1}{p^z}\right)^{-1}\right] \quad (26)$$

Such that (25) becomes:

$$\zeta(z) = \frac{2\pi^{z/2}\frac{z}{2}\Gamma\left({}^{z}/_{2}\right)(z-1)\pi^{-z/2}\prod_p\left(1-\frac{1}{p^z}\right)^{-1}}{z(z-1)\Gamma\left({}^{z}/_{2}\right)} \quad (27)$$

$$\partial_E(z) = \frac{z}{2}(z-1)\Gamma\left({}^{z}/_{2}\right)\left[\frac{z}{\zeta(z)}\int_0^\infty frac\left(\frac{1}{t}\right)t^{z-1}dt\left[\frac{2}{(z-1)}\prod_{i=1}^{k}P_i^{\alpha_i}\left\{\left(zk+\frac{p(t)}{(z-1)}\right)\right\}\right]\right] \quad (22)$$

Or we can write equations (22) and (22a) as;

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and

$$\partial_E(z) = \frac{\Gamma\left({}^{z}/_{2}\right)}{z(z-1)}\left[-\frac{z^2}{2}(z-1)\sum_{n=1}^{\infty}\frac{1}{2^{n-1}}\prod_{i=1}^{k}P_i^{\alpha_i}\left\{\left(zk+\frac{p(t)}{(z-1)}\right)\right\}\right] \quad (22c)$$

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$$\zeta(z) = \frac{z\Gamma(z/2)}{z(z-1)}\prod_p\left(1-\frac{1}{p^z}\right)^{-1}\left[\frac{\frac{2}{2}\pi^{z/2}\pi^{-z/2}(z-1)}{\Gamma(z/2)}\right] \quad (28)$$

$$\zeta(z) = \frac{\Gamma(z/2)}{z(z-1)}\prod_p\left(1-\frac{1}{p^z}\right)^{-1}\left[\frac{z(z-1)}{\Gamma(z/2)}\right] \quad (29)$$

This means that the difference in the work of Euler (1749) and Riemann (1859) is simply;

$$\frac{\Gamma(z/2)}{z(z-1)}\left[\frac{z(z-1)}{\Gamma(z/2)}\right] = 1$$

By comparing (22) and (29), it is obviously obvious that;

$$\prod_p\left(1-\frac{1}{p^z}\right)^{-1}\left[\frac{z(z-1)}{\Gamma(z/2)}\right] \cong -z^2(z-1)\prod_{i=1}^{k}P_i^{\alpha_i}\left\{\left(zk+\frac{p(t)}{(z-1)}\right)\right\} \quad (30)$$

$$\Rightarrow \prod_p\left(1-\frac{1}{p^z}\right)^{-1}\left[\frac{z(z-1)}{\Gamma(z/2)}\right] \cong -\prod_{i=1}^{k} z^2P_i^{\alpha_i}\left(k(z^2-z)+p(t)\right) \quad (31)$$

$$\Rightarrow \frac{\prod_p\left(1-\frac{1}{p^z}\right)^{-1}\left[\frac{z(z-1)}{\Gamma(z/2)}\right]}{\prod_{i=1}^{k} z^2P_i^{\alpha_i}} \cong -\left(k(z^2-z)+p(t)\right) \quad (32)$$

By this, we now obtain a representation for $p(t)$ that allows the transformation of (9) unto the Riemann zeta function such that $p(t)$ is given as;

$$p(t) = -\left[\frac{z(z-1)\prod_p\left(1-\frac{1}{p^z}\right)^{-1}}{z^2\Gamma(z/2)\prod_{i=1}^{k}P_i^{\alpha_i}} + k(z^2-z)\right] \quad (33)$$

Equation (33) implies that if $p(t)$ is substituted into (22), equation (22) will become;

$$\partial_g(z) = \frac{\Gamma(z/2)}{z(z-1)}z^2\prod_{i=1}^{k}P_i^{\alpha_i}\left\{k(z^2-z)+\frac{(z-1)\prod_p\left(1-\frac{1}{p^z}\right)^{-1}}{z\Gamma(z/2)\prod_{i=1}^{k}P_i^{\alpha_i}} - k(z^2-z)\right\} \quad (34)$$

$$\partial_E(z) = \frac{\Gamma(z/2)}{(z-1)} z \prod_{i=1}^{k} P_i^{\alpha_i} \left\{ \frac{(z-1)\prod_p \left(1-\frac{1}{p^z}\right)^{-1}}{z\Gamma(z/2)\prod_{i=1}^{k} P_i^{\alpha_i}} \right\} \quad (35)$$

$$\partial_E(z) = \prod_p \left(1-\frac{1}{p^z}\right)^{-1} \quad (36)$$

Or from (22c), we can see that ;

$$\prod_p \left(1-\frac{1}{p^z}\right)^{-1} \left[\frac{z(z-1)}{\Gamma(z/2)}\right] \cong -\frac{z^2}{2}(z-1)\sum_{n=1}^{\infty}\frac{1}{2^{n-1}}\prod_{i=1}^{k} P_i^{\alpha_i}\left\{\left(zk+\frac{p(t)}{(z-1)}\right)\right\} \quad (36a)$$

$$\frac{2\prod_p \left(1-\frac{1}{p^z}\right)^{-1}}{z\Gamma(z/2)\sum_{n=1}^{\infty}\frac{1}{2^{n-1}}\prod_{i=1}^{k} P_i^{\alpha_i}} \cong -\left\{\left(zk+\frac{p(t)}{(z-1)}\right)\right\} \quad (36b)$$

Thus, equation (14) is the same as (16) and (25) given that $p(t)$ equal to equation (33).

Enoch (2015) also obtained $p(t) = 1 + kt^2$ from numerical experiment such that one arrives at;

$$p(t) = 1 + kt^2 = -\frac{(z-1)\prod_p \left(1-\frac{1}{p^z}\right)^{-1}}{z\Gamma(z/2)\prod_{i=1}^{k} P_i^{\alpha_i}} - k(z^2 - z) \quad (37)$$

Theorem 1:

Let $p(t)$ be defined as it is in equation (36b) and (37) then it can be shown that the RHS =LHS of (37) and that the zeros of the analytic continuation formula, $s(t)$, of the Riemann Zeta Function will always be real as $n \to \infty$

Proof: Let

$$1 + kt^2 = -\frac{(z-1)\prod_p (1-P^{-z})^{-1}}{z\Gamma(z/2)n} - k(z^2 - z) \quad (38)$$

$$t^2 = -\frac{(z-1)\prod_p (1-P^{-z})^{-1}}{kz\Gamma(^z/_2)n} - (z^2 - z) - \frac{1}{k} \quad (39)$$

$$t^2 = \left[-\frac{(z-1)\prod_p (1-P^{-z})^{-1}}{zkn\Gamma(^z/_2)} - z(z-1) - \frac{1}{k}\right] \quad (40)$$

$$t^2 = \left[\frac{-(z-1)\prod_p (1-P^{-z})^{-1} - [z^3k - z^2k + z]\, n\Gamma(^z/_2)}{zkn\Gamma(^z/_2)} - \frac{1}{k}\right] \quad (41)$$

If one takes the limit of (41) as $n \to \infty$ *and* $z = \frac{1}{2} \pm it$, one obtains;

$$\lim_{n\to\infty} t^2 = \pm \lim_{n\to\infty}\left[-\frac{(z-1)\prod_p (1-P^{-z})^{-1}}{zkn\Gamma(^z/_2)} - z(z-1) - \frac{1}{k}\right] \quad (42)$$

$$\lim_{n\to\infty} t^2 = \left[-\frac{(z-1)\prod_p (1-P^{-z})^{-1}}{zk\infty\Gamma(^z/_2)} - \lim_{n\to\infty} z(z-1) - \lim_{n\to\infty}\frac{1}{k}\right] \quad (43)$$

$$\lim_{n\to\infty} t^2 = \left[0 - \lim_{n\to\infty} z(z-1) - \lim_{n\to\infty}\frac{1}{k}\right] \quad (44)$$

$$\lim_{n\to\infty} t = \pm\left[0 - \lim_{n\to\infty} z(z-1) - \lim_{n\to\infty}\frac{1}{k}\right]^{1/2} \quad (45)$$

If we choose $z = \frac{1}{2} \pm it$ *and* $k = 4$ *in* (45) ,we arrival at:

$$\lim_{n\to\infty} t = \pm\left[-\lim_{n\to\infty}\left(\frac{1}{2} \pm it\right)\left(\frac{1}{2} \pm it - 1\right) - \lim_{n\to\infty}\frac{1}{4}\right]^{1/2} \quad (46)$$

The RHS is shown that

$$RHS = \pm\left[-\lim_{n\to\infty}\left(\frac{1}{4} + it - \frac{1}{2} - it - t^2\right) - \lim_{n\to\infty}\frac{1}{4}\right]^{1/2} \quad (47)$$

$$RHS = \pm\left[\lim_{n\to\infty}\frac{1}{4} + \lim_{n\to\infty} t^2 - \lim_{n\to\infty}\frac{1}{4}\right]^{1/2} \quad (41)$$

$$RHS = \pm\left[\lim_{n\to\infty} t^2\right]^{1/2} = \pm t, \qquad t \in \mathbb{R} \quad (42)$$

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Theorem 2: Let the Analytical continuation formula of the Riemann Zeta function be given as:

$$\varepsilon(t) = \frac{1}{2} - \left(t^2 + \frac{1}{4}\right)\int_1^{\infty} \psi(x)x^{-\frac{z}{4}}cos\left(\frac{t}{2}\log x\right)dx;\ \psi(x) = e^{-n^2\pi x}$$

If t is the zero of $\varepsilon(t)$ then it can be shown that $\varepsilon(t) = 0$ anytime

$$t = \pm\left[-\frac{(z-1)\prod_p(1-P^{-z})^{-1}}{4zn\Gamma\left(z/_2\right)} - z(z-1) - \frac{1}{4}\right]^{1/_2}$$

Proof 2: We show that

$$\varepsilon(t) = \frac{1}{2} - \left(-\frac{(z-1)\prod_p(1-P^{-z})^{-1}}{4z\Gamma\left(z/_2\right)n} - (z^2 - z)\right)\int_1^{\infty} e^{-n^2\pi x}x^{-\frac{z}{4}}cos(\varrho\log x)dx \qquad (43)$$

$$Where\ \ \varrho = \frac{1}{2}\left[-\frac{(z-1)\prod_p(1-P^{-z})^{-1}}{4zn\Gamma\left(z/_2\right)} - z(z-1) - \frac{1}{4}\right]^{1/_2}$$

$\varepsilon(t) = 0$Implies that;

$$\frac{1}{2} = \left(\left(-\frac{\prod_p(1-P^{-z})^{-1}}{4z\Gamma\left(z/_2\right)n} - z\right)(z-1)\right)\int_1^{\infty} e^{-n^2\pi x}x^{-\frac{z}{4}}cos(\varrho\log x)dx \qquad (44)$$

It follows that the RHS of (44) equals to;

$$\left(\left(-\frac{\prod_p(1-P^{-z})^{-1}}{4z\Gamma\left(z/_2\right)n^{1-z+z}} - z\right)(z-1)\right)\int_1^{\infty} e^{-n^2\pi x}x^{-\frac{z}{4}}cos(\varrho\log x)dx \qquad (45)$$

This means that (44) can be written as;

$$\frac{1}{2} = \left(\left[-\frac{\prod_p(1-P^{-z})^{-1}}{4z\Gamma\left(z/_2\right)n^{-z}n^{1+z}} - z\right](z-1)\right)\int_1^{\infty} e^{-n^2\pi x}x^{-\frac{z}{4}}cos(\varrho\log x)dx \qquad (46)$$

$$= \left(\frac{(z-1)}{4z\Gamma\left(z/_2\right)\sum_{n\geq1}^{\infty} n^{1+z}} - (z^2 - z)\right)\int_1^{\infty} e^{-n^2\pi x}x^{-\frac{z}{4}}cos\left(\frac{1}{2}\left(\frac{(z-1)}{4z\Gamma\left(z/_2\right)\sum_{n\geq1}^{\infty} n^{1+z}} - (z^2 - z) - \frac{1}{4}\right)^{1/_2}\log x\right)dx \qquad (47)$$

If one takes the limit of equation (47) as $n \to \infty$, one arrivals at ;

$$\lim_{n\to\infty}\frac{1}{2}=\lim_{n\to\infty}\left(\frac{(z-1)}{4z\Gamma(z/2)\sum_{n\geq 1}^{\infty}n^{1+z}}-(z^2-z)\right)\int_1^{\infty}e^{-n^2\pi x}x^{-\frac{3}{4}}cos\left(\frac{1}{2}\left(\frac{(z-1)}{4z\Gamma(z/2)\sum_{n\geq 1}^{\infty}n^{1+z}}-(z^2-z)-\frac{1}{4}\right)^{1/2}\log x\right)dx \quad (48)$$

$$\frac{1}{2}=\left(\frac{1}{4}+t^2\right)\lim_{n\to\infty}\int_1^{\infty}e^{-n^2\pi x}x^{-\frac{3}{4}}cos\left(\frac{1}{2}\left(\frac{(z-1)}{4z\Gamma(z/2)\sum_{n\geq 1}^{\infty}n^{1+z}}-(z^2-z)-\frac{1}{4}\right)^{1/2}\log x\right)dx \quad (49)$$

$$\frac{1}{2}=\left(\frac{1}{4}+t^2\right)\int_1^{\infty}\lim_{n\to\infty}e^{-n^2\pi x}x^{-\frac{3}{4}}cos\left(\frac{1}{2}\lim_{n\to\infty}\left(\frac{(z-1)}{4z\Gamma(z/2)\sum_{n\geq 1}^{\infty}n^{1+z}}-\left(-t^2-\frac{1}{4}\right)-\frac{1}{4}\right)^{1/2}\log x\right)dx \quad (50)$$

It is obviously obvious that the intergrade in (50) implies

$$\int_1^{\infty}\lim_{n\to\infty}e^{-n^2\pi x}x^{-\frac{3}{4}}cos\left(\frac{1}{2}\lim_{n\to\infty}\left(\frac{(z-1)}{4z\Gamma(z/2)\sum_{n\geq 1}^{\infty}n^{1+z}}+t^2\right)^{1/2}\log x\right)dx=\frac{2}{1+4t^2} \quad (51)$$

And with this reality, equation (43) will always be equal to;

$$\varepsilon(t)=\frac{1}{2}-\left(t^2+\frac{1}{4}\right)\left(\frac{2}{1+4t^2}\right)=0;$$

Anytime

$$t=\pm\left[\frac{(z-1)\prod_p(1-P^{-z})^{-1}}{4zn\Gamma(z/2)}-z(z-1)-\frac{1}{4}\right]^{1/2}$$

QED.

Using the Mellin's Transformation

If we make use of

$$\int_0^{\infty}frac\left(\frac{1}{t}\right)t^{z-1}dt=-\frac{\zeta(z)}{z}; for\ \ 0<\mathbb{R}(z)<1, frac\ means\ the\ fractional\ part$$

Such that

$$\frac{z}{\zeta(z)}\int_0^\infty frac\left(\frac{1}{t}\right)t^{z-1}dt = -1; for \ \ 0 < \mathbb{R}(z) < 1$$

It implies from equations (14) and (21) that:

$$\partial_E(z) = z^2\Gamma\left({}^{z}/_{2}\right)\left(\frac{1}{\zeta(z)}\int_0^\infty frac\left(\frac{1}{t}\right)t^{z-1}dt\right)\left[\prod_{i=1}^{k} P_i^{\propto_i}\left\{\left(zk + \frac{p(t)}{(z-1)}\right)\right\}\right] \qquad (22)$$

It also follows from equations (14) and (21b) that:

$$\partial_E(z) = z^2\Gamma\left({}^{z}/_{2}\right)\left(\frac{1}{\zeta(z)}\int_0^\infty frac\left(\frac{1}{t}\right)t^{z-1}dt\right)\left[\sum_{n=1}^{\infty}\frac{1}{2^n}\prod_{i=1}^{k} P_i^{\propto_i}\left\{\left(zk + \frac{p(t)}{(z-1)}\right)\right\}\right] \ (22a)$$

If wesubstitute (30) into (30), we will obtain;

$$\Rightarrow\left(\frac{z}{\zeta(z)}\int_0^\infty frac\left(\frac{1}{t}\right)t^{z-1}dt\right)\prod_p\left(1-\frac{1}{p^z}\right)^{-1}\left[\frac{1}{z\Gamma\left({}^{z}/_{2}\right)}\right] \cong \prod_{i=1}^{k} P_i^{\propto_i}\left\{\left(zk + \frac{p(t)}{(z-1)}\right)\right\} \qquad (31)$$

On further simplification of (22) or (22a), we obtain;

$$\partial_E(z) = z^2\Gamma\left({}^{z}/_{2}\right)\left(\frac{1}{\zeta(z)}\int_0^\infty frac\left(\frac{1}{t}\right)t^{z-1}dt\right)\left(\frac{1}{\Gamma\left(\frac{z}{2}\right)}\int_0^\infty frac\left(\frac{1}{t}\right)t^{z-1}dt\right) \qquad (22)$$

Such that (22)

$$\partial_E(z) = z^2\left(-\frac{1}{\zeta(z)}\frac{\zeta(z)}{z}\right)\left(-\frac{\zeta(z)}{z}\right) \qquad (22)$$

And eventually, we obtain the desired transformation;

$$\partial_E(z) = \zeta(z) \qquad (22)$$

Conclusion and Acknowledgments

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