Vol. 3, No 1 June 2020
ISSN: 2016-1303 | ***Web:*** *www.cjpas.fulokoja.edu.ng*

AN ORTHOGONAL POLYNOMIAL AS A BASIS FUNCTION FOR THE FORMULATION OF HYBRID METHODS FOR FIRST ORDER INITIAL VALUE PROBLEM

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ABSTRACT
A class of orthogonal polynomial was constructed in the interval [0, 1], with weight function w(x) = 1 - x which was employed as a basis function of the approximant to the solution of first Order ordinary differential equation, for generating the required numerical scheme through collocation and interpolation approach. Some classes of continuous linear multistep method for the solution of the problem were derived. The resulting scheme were implemented on some examples, the result show that the method is effective.

Keywords: Orthogonal Polynomials, Ordinary differential equations, Continuous Scheme, Explicit method, hybrid method.

1.0 INTRODUCTION
In science and engineering, usually mathematical models are developed to help in the understanding of physical phenomena. These models often yield equations that contain some derivatives of an unknown function of one or several variables. Such equations are called differential equations. Differential equations do not only arise in the physical sciences but also in diverse fields as economics, medicine, psychology, Operation research and even in areas such as biology and anthropology. Interestingly, differential equations arising from the modeling of physical phenomena, often do not have analytic solutions. Hence, the development of numerical methods to obtain approximate solutions becomes necessary. To that extent, several numerical methods such as finite difference methods, finite element methods and finite volume methods, among others, have been developed based on the nature and type of the differential equation to be solved. A differential equation in which the unknown function is a function of two or more independent variable is called partial differential equation. Those in which the unknown function is a function of only one independent variable are called ordinary differential equations. This work concerns the study of numerical solutions of the latter. This article focuses on continuous hybrid method for the solution of first order ordinary differential equation of the form.

$$y'(x) = f\big(x, y(x)\big), y(x_0) = y_0 \qquad 1.0$$

The techniques used in the derivation of Linear Multistep Methods (LMMs) using collocation approach for the numerical solution of the IVPs (1.0) have been reported in the literature [see [1], [2], [3], [5] and [8] to mention few. The method used to solve (1.0) vary according to the type of bases functions involved in the approximant *Y(x) of y(x)* Among the techniques are Backward differentiation methods, Adams Moulton methods, Adams Bashforth methods, collocation methods and so on. Great effort has been made in research on the continuous formulation of discrete initial value problems which has the ability of yielding several output at the off grid points without requiring additional interpolation when compared with their discrete equivalent.

Vol. 3, No 1 June 2020
ISSN: 2016-1303 | ***Web:*** *www.cjpas.fulokoja.edu.ng*

This research work employs a collocation approach for the formulation of continuous hybrid method using a set of orthogonal polynomials with $\varphi(x) = 1 - x$ over an interval [0, 1] as bases function.
This article consist of a number of section. From section 2.0 to section 5.0, we shall present the construction of our proposed numerical schemes for problem (1.0). Section 6.0 provides an analysis for the derived schemes, section 7.0 illustrates the methods using some selected test problems. Finally, the paper is ended in section 8.0 with some concluding remarks.

2.0 MATERIALS AND METHODS

Let's first consider the derivation of the orthogonal polynomials with weight function $\varphi(x) = 1 - x$ valid in the interval [0, 1].

Let $P_n(x) = \sum_{r=0}^{n} C_r^n * x^r$ be a polynomial of order nth. The requirement for the construction is that:

$$P_n(1) = 1 \qquad 2.0$$

$$\int_0^1 \varphi(x) * P_n(x) * P_m(x)dx = 0 \qquad 2.1$$

For $n = 1$, we have

$$P_1(x) = \sum_{r=0}^{1} C_r^1 * x^r = C_0^1 + C_1^1 x \qquad 2.2$$

From (2.0) and (2.1) we have,

$$P_1(x) = \sum_{r=0}^{1} C_r^1 * x^r \Rightarrow C_0^1 + C_1^1 x = 1 \qquad 2.3$$

$$\int_0^1 \varphi(x) * P_1(x) * P_0(x)dx \Rightarrow \frac{1}{6}C_1^1 + \frac{1}{2}C_0^1 = 0 \qquad 2.4$$

Where $P_0(x) = 1$, solving (2.3) and (2.4) simultaneously yield $C_0^1 = -\frac{1}{2}$ and $C_1^1 = \frac{3}{2}$

Which when substituted into (2.2) yield $P_1(x) = -\frac{1}{2} + \frac{3}{2}x$.

Similarly for $n = 2$, we have

$$P_2(x) = \sum_{r=0}^{2} C_r^2 * x^r = C_0^2 + C_1^2 x + C_1^2 x^2 \qquad 2.5$$

Then from (2.0) and (2.1) we have

$$P_2(x) = \sum_{r=0}^{2} C_r^2 * x^r = 1 \qquad 2.6$$

$$\int_0^1 \varphi(x) * P_2(x) * P_0(x)dx = 0 \qquad 2.7$$

$$\int_0^1 \varphi(x) * P_2(x) * P_1(x)dx = 0 \qquad 2.8$$

Solving (2.6), (2.7) and (2.8) simultaneously and substituting their values into (2.5) yield $P_2(x) = \frac{1}{3} - \frac{8}{3}x + \frac{10}{3}x^2$. By so doing we developed orthogonal polynomials from $p_0(x)$ to $p_7(x)$.

Hence from equation (2.0) and (2.1), we have the following first seven orthogonal polynomials

$$p_0(x) = 1$$

$$P_1(x) = -\frac{1}{2} + \frac{3}{2}x$$

$$P_2(x) = \frac{1}{3} - \frac{8}{3}x + \frac{10}{3}x^2$$

$$P_3(x) = -\frac{1}{4} + \frac{15}{4}x - \frac{45}{4}x^2 + \frac{35}{4}x^3$$

$$P_4(x) = \frac{1}{5} - \frac{24}{5}x + \frac{126}{5}x^2 - \frac{224}{5}x^3 + \frac{126}{5}x^4$$

$$P_5(x) = -\frac{1}{6} + \frac{35}{6}x - \frac{140}{3}x^2 + 140x^3 - 175x^4 + 77x^5$$

$$P_6(x) = \frac{1}{7} - \frac{48}{7}x + \frac{540}{7}x^2 - \frac{2400}{7}x^3 + \frac{4950}{7}x^4 - \frac{4752}{7}x^5 + \frac{1716}{7}x^6$$

$$P_7(x) = -\frac{1}{8} + \frac{63}{8}x - \frac{945}{8}x^2 + \frac{5775}{8}x^3 - \frac{17325}{8}x^4 + \frac{27027}{8}x^5 - \frac{21021}{8}x^6 + \frac{6435}{8}x^7$$

Which are employed as basis function in the derivation of the hybrid methods.
The transformation $x = aX + b$ was used and varies as the step number varies. So, we shall now seek the solution of the problem:

$$y'(x) = f(x, y(x)), \ x_n \le x \le x_{n+q} \qquad 2.2$$

$$y(x_n), = y_n \qquad 2.3$$

$$x_n = x_0 + nh, \ h = \frac{x_n - x_0}{h} \qquad 2.4$$

$$Y(x) = \sum_{r=0}^{n} a_r * p_r(x) \cong y(x) \qquad 2.5$$

Vol. 3, No 1 June 2020
ISSN: 2016-1303 | ***Web:*** *www.cjpas.fulokoja.edu.ng*

2.1 Three Step method with two off grid point

Considering equation (2.5) at $n = 6$

$$Y(x)=a_0+a_1\left(-\frac{1}{2}+\frac{3}{2}x\right)+a_2\left(\frac{1}{3}-\frac{8}{3}x+\frac{10}{3}x^2\right)+a_3\left(-\frac{1}{4}+\frac{15}{4}x-\frac{45}{4}x^2+\frac{35}{4}x^3\right)$$
$$+a_4\left(\frac{1}{5}-\frac{24}{5}x+\frac{126}{5}x^2-\frac{224}{5}x^3+\frac{126}{5}x^4\right)+$$
$$a_5\left(-\frac{1}{6}+\frac{35}{6}x-\frac{140}{3}x^2+140x^3-175x^4+77x^5\right)$$
$$+a_6\left(\frac{1}{7}-\frac{48}{7}x+\frac{540}{7}x^2-\frac{2400}{7}x^3+\frac{4950}{7}x^4-\frac{4752}{7}x^5+\frac{1716}{7}x^6\right) \quad 3.0$$

Evaluating (3.0) at $x=\frac{x-x_n}{h}$ and then interpolating and collocating at x_n and $x_{n+1}, x_{n+\frac{3}{2}}, x_{n+2}$, $x_{n+\frac{5}{2}}$ and x_{n+3} yield the following system of equations:

$$a_0-\frac{1}{2}a_1+\frac{1}{3}a_2-\frac{1}{4}a_3+\frac{1}{5}a_4-\frac{1}{6}a_5+\frac{1}{7}a_6=y_n$$

$$\frac{3}{2h}a_1-\frac{8}{3h}a_2+\frac{15}{4h}a_3-\frac{24}{5h}a_4+\frac{35}{6h}a_5-\frac{48}{7h}a_6=f_n$$

$$\frac{3}{2h}a_1+\frac{4}{h}a_2+\frac{15}{2h}a_3+\frac{12}{h}a_4+\frac{35}{2h}a_5+\frac{24}{h}a_6=f_{n+1}$$

$$\frac{3}{2h}a_1+\frac{22}{3h}a_2+\frac{465}{16h}a_3+\frac{543}{5h}a_4+\frac{19075}{48h}a_5+\frac{40389}{28h}a_6=f_{n+\frac{3}{2}}$$

$$\frac{3}{2h}a_1+\frac{32}{3h}a_2+\frac{255}{4h}a_3+\frac{1824}{5h}a_4+\frac{12355}{6h}a_5+\frac{81024}{7h}a_6=f_{n+2}$$

$$\frac{3}{2h}a_1+\frac{14}{h}a_2+\frac{1785}{16h}a_3+\frac{4281}{5h}a_4+\frac{103985}{16h}a_5+\frac{1377483}{28h}a_6=f_{n+\frac{5}{2}}$$

$$\frac{3}{2h}a_1+\frac{8}{3h}a_2+\frac{15}{4h}a_3+\frac{8292}{5h}a_4+\frac{94745}{6h}a_5+\frac{1050360}{7h}a_6=f_{n+3}$$

Solving this system of equations by Gaussian elimination method gives

$$a_0=\frac{97}{168}hf_{n+1}-\frac{2}{7}hf_{n+\frac{5}{2}}+\frac{659}{3780}hf_n-\frac{118}{135}hf_{n+\frac{3}{2}}+\frac{97}{140}hf_{n+2}+\frac{367}{7560}hf_{n+3}+y_n$$

$$a_1=\frac{1367}{945}hf_{n+1}-\frac{3016}{4725}hf_{n+\frac{5}{2}}+\frac{2998}{14175}hf_n-\frac{5752}{2835}hf_{n+\frac{3}{2}}+\frac{494}{315}hf_{n+2}+\frac{61}{567}hf_{n+3}$$

$$a_2=\frac{59}{560}hf_{n+1}-\frac{3967}{25200}hf_n+\frac{58}{315}hf_{n+\frac{3}{2}}+\frac{18}{175}hf_{n+\frac{5}{2}}-\frac{121}{560}hf_{n+2}+\frac{19}{1008}hf_{n+3}$$

$$a_3=-\frac{3662}{10395}hf_{n+1}+\frac{116}{2079}hf_n+\frac{6304}{10395}hf_{n+\frac{3}{2}}+\frac{1952}{10395}hf_{n+\frac{5}{2}}-\frac{1616}{3465}hf_{n+2}-\frac{326}{10395}hf_{n+3}$$

$$a_4=\frac{3445}{33264}hf_{n+1}-\frac{335}{33264}hf_n-\frac{470}{2079}hf_{n+\frac{3}{2}}-\frac{190}{2079}hf_{n+\frac{5}{2}}+\frac{2305}{11088}hf_{n+2}+\frac{535}{33264}hf_{n+3}$$

$$a_5=-\frac{58}{5005}hf_{n+1}+\frac{8}{9009}hf_n+\frac{184}{6435}hf_{n+\frac{3}{2}}+\frac{72}{5005}hf_{n+\frac{5}{2}}-\frac{148}{5005}hf_{n+2}-\frac{122}{45045}hf_{n+3}$$

$$a_6=\frac{7}{15444}hf_{n+1}-\frac{7}{231660}hf_n-\frac{14}{11583}hf_{n+\frac{3}{2}}-\frac{14}{19305}hf_{n+\frac{5}{2}}+\frac{7}{5148}hf_{n+2}+\frac{7}{46332}hf_{n+3}$$

Substituting this values into (2.6) yield the continuous scheme in the form of

$$Y(x)=y_n+h\left[\beta_0(x)f_n+\beta_1(x)f_{n+1}+\beta_{\frac{3}{2}}(x)f_{n+\frac{3}{2}}+\beta_2(x)f_{n+2}+\beta_{\frac{5}{2}}(x)f_{n+\frac{5}{2}}+\beta_3(x)f_{n+3}\right] \quad 3.1$$

Where β_0, β_1, $\beta_{\frac{3}{2}}$, β_2, $\beta_{\frac{5}{2}}$ and β_3 are the coefficient of f_n, f_{n+1}, $f_{n+\frac{3}{2}}$, f_{n+2}, $f_{n+\frac{5}{2}}$ and f_{n+3} when all the values of a_n are substituted into (3.0) where $n = 1,2,3,4,5,6$.

At the grid point x_{n+1}, x_{n+2} *and* x_{n+3} yield the following hybrid scheme:

$$y_{n+1}=y_n+\frac{11h}{40}\left(f_n+\frac{673}{99}f_{n+1}-\frac{832}{99}f_{n+\frac{3}{2}}+\frac{211}{33}f_{n+2}-\frac{256}{99}f_{n+\frac{5}{2}}+\frac{43}{99}f_{n+3}\right) \quad 3.2$$

$$y_{n+2}=y_n+\frac{37h}{135}\left(f_n+\frac{276}{37}f_{n+1}-\frac{224}{37}f_{n+\frac{3}{2}}+\frac{261}{37}f_{n+2}-\frac{96}{37}f_{n+\frac{5}{2}}+\frac{16}{37}f_{n+3}\right) \quad 3.3$$

$$y_{n+3} = y_n + \frac{11h}{40}\left(f_n + \frac{81}{11}f_{n+1} - \frac{64}{11}f_{n+\frac{3}{2}} + \frac{81}{11}f_{n+2} - \frac{96}{37}f_{n+\frac{5}{2}} + f_{n+3}\right) \quad 3.4$$

To determine $f_{n+\frac{3}{2}}$ and $f_{n+\frac{5}{2}}$ in equation (3.2), (3.3) and (3.4), we evaluate (3.0) at $x = \frac{x-x_n}{h}$ and then interpolate and collocate at x_n, x_{n+1}, x_{n+2} and $x_n, x_{n+1}, x_{n+2}, x_{n+3}$ to have

$$a_0 - \frac{1}{2}a_1 + \frac{1}{3}a_2 - \frac{1}{4}a_3 + \frac{1}{5}a_4 - \frac{1}{6}a_5 + \frac{1}{7}a_6 = y_n$$

$$a_0 + a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = y_{n+1}$$

$$a_0 + \frac{5}{2}a_1 + \frac{25}{3}a_2 + \frac{129}{4}a_3 + \frac{681}{5}a_4 + \frac{3653}{6}a_5 + \frac{19825}{7}a_6 = y_{n+2}$$

$$\frac{3}{2h}a_1 - \frac{8}{3h}a_2 + \frac{15}{4h}a_3 - \frac{24}{5h}a_4 + \frac{35}{6h}a_5 - \frac{48}{7h}a_6 = f_n$$

$$\frac{3}{2h}a_1 + \frac{4}{h}a_2 + \frac{15}{2h}a_3 + \frac{12}{h}a_4 + \frac{35}{2h}a_5 + \frac{24}{h}a_6 = f_{n+1}$$

$$\frac{3}{2h}a_1 + \frac{32}{3h}a_2 + \frac{255}{4h}a_3 + \frac{1824}{5h}a_4 + \frac{12355}{6h}a_5 + \frac{81024}{7h}a_6 = f_{n+2}$$

$$\frac{3}{2h}a_1 + \frac{52}{3h}a_2 + \frac{345}{2h}a_3 + \frac{8292}{5h}a_4 + \frac{94745}{6h}a_5 + \frac{1050360}{7h}a_6 = f_{n+3}$$

After solving the above system of equation to obtained the values of the unknown constant and inserting these values into equation (3.0) yield the following continuous scheme:

$$Y(x) = \alpha_0 y_n + \alpha_0 y_{n+1} + \alpha_0 y_{n+2} + h[\beta_0(x)f_n + \beta_1(x)f_{n+1} + \beta_2(x)f_{n+2} + \beta_3(x)f_{n+3}] \quad 3.5$$

Which at the off grid point $x_{n+\frac{3}{2}}, x_{n+\frac{5}{2}}$ gives a methods

$$y_{n+\frac{3}{2}} = \frac{13}{256}y_n + \frac{189}{352}y_{n+1} + \frac{1161}{2816}y_{n+2} + \frac{7h}{704}\left(f_n + \frac{69}{4}f_{n+1} - \frac{15}{2}f_{n+2} + \frac{1}{12}f_{n+3}\right)$$

2.2 Two Step method with two off grid point

Considering equation (2.5) at $n = 5$. i.e.

$$Y(x) = a_0 + a_1\left(-\frac{1}{2}+\frac{3}{2}x\right) + a_2\left(\frac{1}{3}-\frac{8}{3}x+\frac{10}{3}x^2\right) + a_3\left(-\frac{1}{4}+\frac{15}{4}x-\frac{45}{4}x^2+\frac{35}{4}x^3\right)$$
$$+a_4\left(\frac{1}{5}-\frac{24}{5}x+\frac{126}{5}x^2-\frac{224}{5}x^3+\frac{126}{5}x^4\right)$$
$$+a_5\left(-\frac{1}{6}+\frac{35}{6}x-\frac{140}{3}x^2+140x^3-175x^4+77x^5\right)$$

By following similar procedure as in above, we interpolate at $x = x_n$ and collocate at x_n, $x_{n+\frac{1}{2}}, x_{n+1}, x_{n+\frac{3}{2}}$ and x_{n+2}.

Evaluating at different grids and off-grid point

$$y_{n+1} = y_n + \frac{29h}{180}\left(\frac{4}{29}f_{n+\frac{3}{2}} + \frac{124}{29}f_{n+\frac{1}{2}} + \frac{24}{29}f_{n+1} + f_n - \frac{1}{29}f_{n+3}\right) \quad 4.0$$

$$y_{n+2} = y_n + \frac{7h}{45}\left(\frac{32}{7}f_{n+\frac{3}{2}} + \frac{32}{7}f_{n+\frac{1}{2}} + \frac{12}{7}f_{n+1} + f_n + f_{n+3}\right) \quad 4.1$$

$$y_{n+\frac{1}{2}} = \frac{45}{128}y_n + \frac{9}{16}y_{n+1} + \frac{11}{128}y_{n+2} + \frac{9h}{128}\left(-4f_{n+1} + f_n - \frac{1}{3}f_{n+2}\right) \quad 4.2$$

$$y_{n+\frac{3}{2}} = \frac{11}{128}y_n + \frac{9}{16}y_{n+1} + \frac{45}{128}y_{n+2} + \frac{3h}{128}(12f_{n+1} + f_n - 3f_{n+2}) \quad 4.3$$

2.3 One Step method with two off grid point

Considering equation (2.5) at $n = 3$.

Following similar procedure, we interpolate at $x = x_n$ and collocate at x_n, $x_{n+\frac{4}{5}}$ and $x_{n+\frac{9}{10}}$

Evaluating at different grids and off-grid points, yield the following schemes:

$$y_{n+1} = y_n + \frac{61h}{216}\left(-\frac{160}{61}f_{n+\frac{9}{10}} + \frac{315}{61}f_{n+\frac{4}{5}} + f_n\right) \quad 5.0$$

$$y_{n+\frac{4}{5}} = \frac{13}{125}y_n + \frac{112}{125}y_{n+1} + \frac{h}{125}(-16f_{n+1} + 4f_n) \qquad 5.1$$

$$y_{n+\frac{9}{10}} = \frac{7}{250}y_n + \frac{243}{250}y_{n+1} + \frac{9h}{1000}(-9f_{n+1} + f_n) \qquad 5.2$$

2.4 Discussion of Consistency, Zero Stability and Convergence of the Methods

Definition:

Zero Stability: A linear multi step method

$\sum_{j=0}^{k}\alpha_j y_{n+j} = h\sum_{j=0}^{k}\beta_j f_{n+j} + h\beta_v f_{n+v}$ is said to be zero stable if no root of it first characteristic polynomial $\mu(\varepsilon)$ has modulus greater than one. Where

$$\mu(r) = \sum_{j=0}^{k}\alpha_j y_{n+j} \qquad 6.0$$

The methods (3.2), (3.3), (3.4), (4.0), (4.1) and (5.0) has $r = 1, \pm 1$, ± 1, 1 and ± 1 which shows zero stability of the methods.

Consistency: a linear multi-step method $\sum_{j=0}^{k}\alpha_j y_{n+j} = h\sum_{j=0}^{k}\beta_j f_{n+j} + h\beta_v f_{n+v}$ is said to be consistence if the following condition are satisfied

I. It has order $p \geq 1$

II. $\mu(1) = 0$ and

III. $\mu'(1) = \emptyset(1)$, where $\emptyset(r) = h\sum_{j=0}^{k}\beta_j f_{n+j} + h\beta_v f_{n+v}$

Table 1: Features of the three step method with two off-grid point.

	Order	Error constant	$\mu(1)$	$\mu'(1)$	$\emptyset(1)$
y_{n+1}	6	$\frac{-47}{24192}$	0	1	1
y_{n+2}	6	$\frac{-29}{15120}$	0	2	2
y_{n+3}	6	$\frac{-9}{440}$	0	3	3
$y_{n+3/2}$	6	$\frac{-39}{788480}$	-	-	-
$y_{n+5/2}$	6	$\frac{-85}{15157696}$	-	-	-

Table 2: Features of the two step method with two off grid points

	Order	Error Constant	$\mu(1)$	$\mu'(1)$	$\emptyset(1)$
y_{n+1}	5	$\frac{1}{5760}$	0	1	1
y_{n+2}	6	$\frac{-1}{15120}$	0	2	2
$y_{n+1/2}$	5	$\frac{1}{5120}$	-	-	-
$y_{n+3/2}$	5	$\frac{1}{5120}$	-	-	-

Table 3: Features of the one step method

	Order	Error Constant	$\mu(1)$	$\mu'(1)$	$\emptyset(1)$
y_{n+1}	3	$\frac{-13}{1800}$	0	1	1
$y_{n+4/5}$	3	$\frac{2}{1875}$	-	-	-
$y_{n+9/10}$	3	$\frac{27}{80000}$	-	-	-

Vol. 3, No 1 June 2020
ISSN: 2016-1303 | ***Web:*** *www.cjpas.fulokoja.edu.ng*

From the tables above, the conditions for consistency are satisfied, hence the derived scheme are consistent.

Convergence (Lambert, 1973): The necessary and sufficient condition for a LMM to be convergent are that it must be consistent and zero stable. The discussion above shows that all these methods are convergent since zero stability and consistence conditions are satisfied

1.0 Numerical Examples

The following three (3) problems with their theoretical solutions shall be solved using the new derived schemes.

Problem 1

We consider the problem $y' + y = 0, y(0) = 1$, $h = 0.1$ with exact solution as $y(x) = exp(-x)$.

Table 4: Solutions for Problem 1

x	exact	3S2OGP	2S2OGP	1S2OGP
0.0	1.000000000	1.000000000	1.000000000	1.000000000
0.1	0.904837418	0.9048374179	0.9048374179	0.9053198917
0.2	0.818730753	0.8187307529	0.8187307531	0.8196041063
0.3	0.740818220	0.7408182205	0.7408182206	0.7420039008
0.4	0.670320046	0.6703200458	0.6703200461	0.6717508911
0.5	0.606530659	0.6065306595	0.6065306597	0.6081494440
0.6	0.548811636	0.5488116359	0.5488116362	0.5505697888
0.7	0.496575303	0.4965853035	0.4965853038	0.4984417816
0.8	0.449328964	0.4493289639	0.4493289642	0.4512492598
0.9	0.406569590	0.4065696595	0.4065696598	0.4085249310
1.0	0.367879441	0.3678794409	0.3678794413	0.3698457463

Table 5: Errors for problem 1

x	3S2OGP	2S2OGP	1S2OGP
0.0	0	0	0
0.1	1.00E-10	1.00E-10	4.82E-04
0.2	1.00E-10	1.00E-10	8.73E-04
0.3	5.00E-10	6.00E-10	1.19E-03
0.4	2.00E-10	1.00E-10	1.43E-03
0.5	5.00E-10	7.00E-10	1.62E-03
0.6	1.00E-10	2.00E-10	1.76E-03
0.7	1.00E-05	1.00E-05	1.87E-03
0.8	1.00E-10	2.00E-10	1.92E-03
0.9	6.95E-08	6.98E-08	1.96E-03

Problem 2, we consider the problem $y' - 2y = 0, y(0) = 1$, $h = 0.1$ with exact solution as $y(x) = exp(2x)$.

Table 6: Solutions for Problem 2

x	exact	3S2OGP	2S2OGP	1S2OGP
0.0	1.000000000	1.000000000	1.000000000	1.000000000
0.1	1.221402758	1.221402798	1.221402741	1.223160747
0.2	1.491824698	1.491824745	1.491824690	1.496122213
0.3	1.822118800	1.822118860	1.822118766	1.829997963
0.4	2.225540928	2.225541073	2.225540906	2.238381675
0.5	2.718281828	2.718282004	2.718281763	2.737900601
0.6	3.320116923	3.320117140	3.320116872	3.348892544
0.7	4.055199967	4.055200363	4.055199848	4.096233905
0.8	4.953032424	4.953032907	4.953032323	5.010352522
0.9	6.049647464	6.049648059	6.049647256	6.128466532
1.0	7.389056099	7.389057064	7.389055910	7.496099700

Table 7: Errors for problem 2

x	3S2OGP	2S2OGP	1S2OGP
0.0	0	0	0
0.1	4.00E-08	1.70E-08	1.76E-03
0.2	4.70E-08	8.00E-09	4.30E-03
0.3	6.00E-08	3.40E-08	7.88E-03
0.4	1.45E-07	2.20E-08	1.28E-02
0.5	1.76E-07	6.50E-08	1.96E-02
0.6	2.17E-07	5.10E-08	2.88E-02
0.7	3.96E-07	1.19E-07	4.10E-02
0.8	4.83E-07	1.01E-07	5.73E-02
0.9	5.95E-07	2.08E-07	7.88E-02
1.0	9.65E-07	1.89E-07	1.07E-01

Problem 3

We consider the problem $y' - xy = 0$, $y(0) = 1$, $h = 0.01$ with exact solution as $y(x) = exp\left(\frac{x^2}{2}\right)$. The

Table 8: Solutions for Problem 3

x	Exact	3S2OGP	2S2OGP	1S2OGP
0.00	1.000000000	1.000000000	1.00000000	1.00000000
0.01	1.000050001	1.000100005	1.000100005	1.000100006
0.02	1.000200020	1.000200020	1.000200020	1.000300048
0.03	1.000450101	1.000300045	1.000500125	1.000600188
0.04	1.000800320	1.000700245	1.000800320	1.001000518
0.05	1.001250782	1.001100605	1.001300845	1.001501158
0.06	1.001801621	1.001501126	1.001801621	1.002102260
0.07	1.002453004	1.002202422	1.002503128	1.002804006
0.08	1.003205125	1.002904210	1.003205125	1.003606608
0.09	1.004058212	1.003606488	1.004108416	1.004510308
0.10	1.005012521	1.004610596	1.005012520	1.005515380
0.11	1.006068338	1.005615710	1.005917438	1.006521457

Table 9: Errors for problem 3

x	3S2OGP	2S2OGP	1S2OGP
0.00	0	0	0
0.01	5.00E-05	5.00E-05	5.00E-05
0.02	0.00E+00	0.00E+00	1.00E-04
0.03	1.50E-04	5.00E-05	1.50E-04
0.04	1.00E-04	0.00E+00	2.00E-04
0.05	1.50E-04	5.01E-05	2.50E-04
0.06	3.00E-04	0.00E+00	3.01E-04
0.07	2.51E-04	5.01E-05	3.51E-04
0.08	3.01E-04	0.00E+00	4.01E-04
0.09	4.52E-04	5.02E-05	4.52E-04
0.10	4.02E-04	1.00E-09	5.03E-04
0.11	4.53E-04	1.51E-04	4.53E-04

4.0 Discussion and Conclusion

A method for the formulation of both continuous and discrete multistep scheme for solving IVPs in ODEs has been presented. For this purpose, orthogonal polynomial has been employed as the basis function and a collocation approach was adopted.

The continuous schemes offer several output of solution and agreed with their corresponding discrete schemes at the grid points as could be observed in the tables constructed.
They are desirable as they exhibit the features of efficiency since they require no additional interpolation to yield many results at the off grid points and at no extra cost.
The accuracy of the schemes can also be evidently seen in the numerical examples.

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