MATCHING PAIRS OF DOT PRODUCT OF 7 x 7 ORDER MAGIC SQUARE USING LOUBERE AND MEZERIAC METHODS

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ABSTRACT
Magic square is an interesting part of Mathematics. It's well known unique properties have made it so special that many researchers have discovered more new properties. The special relationship between the sums of the numbers in the square is actually the main factors for which the square is called magic. The concept of magic square has been reviewed. Many orders have been studied. This research work is centered on the matching pairs of the dot products of magic square of order seven using De La Loubere and Bachet De Mezeriac methods. The results showed the same pattern of matching pairs, but Loubere's method is easier to construct and is more general than Mezeriac method.

Keywords: magic square, dot product, magic sum, magic constant.

1.0 INTRODUCTION

By definition, magic square is an arrangement of integers in a square of an order in which the sum of every integer in each column, row and main diagonal is the same (Anwar, M. 2014). Each of these integers will only occur once in the square. The constant sum is called the magic sum or magic constant. If the entries are consecutive integers from 1 *to* n^2, the square is said to be of order n.

Magic squares have been around for over 4,000 years and have been used in different cultures. However, one of the earliest discoveries of magic squares was in China during the Xia dynasty. Indian and Egyptian cultures engraved magic squares onto stones or metal into gowns worn by talismans. They were also used by Arab astrologers in 9th century to help work out horoscopes. In India, magic squares are used in music composition (Laposky, B. F. 1978). The numbers in the square are replaced with musical notes and this can be applied to time cycles and additive rhythm (Ward, W. P. 1991).

There are many ways of producing magic squares. Several construction methods exist. Squares of odd order have different construction method from squares of even order. Several methods of constructing odd order magic square exist (Chee, P. S. 1981). Some of them are: De La Loubere, Bachet De Mezeriac, Phillip la hire, Pherus, Uniform step method and so on. The first three deal with the construction of normal magic square of order n (Sayles, H. 1913). But for the purpose of this research work, we will consider only two methods which are De La Loubere and Bachet De Mezeriac methods.

Definition 1: Square of order n: Square compose of n rows, n columns and n^2 cells,

Definition 2: Dot product: The dot product or inner product of two rows (columns) p and q is obtained by multiplying corresponding elements of P and q summing the results (Dudeney, H. E. 1917). For example, for a square A, the dot product of rows R_p and P_q is given by

$$R_p . R_q = \sum_{i=1}^{n} a_{pi} . a_{qi}$$

and for the product of two column Cp and Cq is given by

$$C_p . C_q = \sum_{i=1}^{n} a_{pi} . a_{qi}$$

Definition 3: Magic sum or magic constant: The constant sum of the n cell of each row, column and main diagonal. E.g for a normal magic square, the sum S is given by

$$S = \frac{n(n^2+1)}{2}$$

Where n is the order of the magic square.

2.0 MATERIAL AND METHODS

For a 7x7 order magic square using the formula

$$\frac{n(n^2+1)}{2}, n = 7$$

(Where n is the number of rows or columns), the magic sum is calculated as follow:

$$S = \frac{n(n^2+1)}{2} = \frac{7(7^2+1)}{2} = \frac{7(49+1)}{2} = 175$$

Below is the method for constructing Loubere magic square:

1. Place the number 1 in the center cell on the top row.
2. Place the next number which is 2 in the bottom row, one column to the right of middle row.
3. Enter the number in sequence in an upward diagonal until it reaches the last column.
4. If encounter an occupied cell then the number is entered into the cell directly below the previously entered cell.
5. Then repeat step 4 and fill in accordingly by the conditions given.

Table 1.1

			1			

Table 1.2

			1			
						4
					3	
				2		

Table 1.3

			1			
		7				
	6					
5						
						4
					3	
				2		

Table 1.4

			1	10		
		7	9			
	6	8				
5						
						4
					3	
				2		

Table 1.5

			1	10	19	
		7	9	18		
	6	8	17			
5	14	16				
13	15					4
					3	12
				2	11	

Table 1.6

			1	10	19	28
		7	9	18	27	29
	6	8	17	26		
5	14	16	25			
13	15	24				4
21	23				3	12
22				2	11	20

Below are the methods for constructing Mezeriac square:
Using same formula $\frac{n(n^2+1)}{2}, n = 7$ for a 7x7 order magic square, the magic sum **S = 175**.

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1. Place 1 to the right of the center cell of 7x7 square.
2. Fill in cells by advancing diagonally upwards to the right until blocked by a previous number.
3. Move two cells left, when a block is encountered.
4. Repeat the process until the square is filled.

Table 2.1

				1		

Table 2.2

						3
					2	
				1		

Table 2.3

4						
						3
					2	
				1		

Table 2.4

4						
						3
					2	
				1		
			7			
		6				
	5					

Table 2.5

4		12				
	11					3
10					2	
				1		9
			7		8	
		6				
	5					

Table 2.6

4		12				
	11					3
10					2	
				1		9
			7		8	
		6		14		15
	5		13			

Table 2.7

4	29	12	37	20	45	28
35	11	36	19	44	27	3
10	42	18	43	26	2	34
41	17	49	25	1	33	9
16	48	24	7	32	8	40
47	23	6	31	14	39	15
22	5	30	13	38	21	46

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3.0 RESULTS AND DISCUSSION

Construction of the dot product of Loubere magic square of order 7x7 above:

The dot product of two rows (columns) p and q is obtained by multiplying corresponding elements of p and q and summing the results. That is, for the magic square A, the dot product of the dot product of rows ***Rp*** ans ***Pq*** is given by:

$$R_p.R_q = \sum_{i=1}^{n} a_{pi}.a_{qi}$$

and for the product of two column ***Cp*** and ***Cq*** is given by:

$$C_p.C_q = \sum_{i=1}^{n} a_{pi}.a_{qi}$$

Finding the dot product of rows from Table 1.7 above, compute:

$R_1.R_2 = (30.38) + (39.47) + (48.7) + (1.9) + (10.18) + (19.27) + (28.29) = 4823$

$R_1.R_3 = (30.46) + (39.6) + (48.8) + (1.17) + (10.26) + (19.35) + (28.37) = 3976$

$R_1.R_4 = (30.5) + (39.14) + (48.16) + (1.25) + (10.34) + (19.36) + (28.45) = 3773$

$R_1.R_5 = (30.13) + (39.15) + (48.24) + (1.33) + (10.42) + (19.44) + (28.4) = 3528$

$R_1.R_6 = (30.21) + (39.23) + (48.32) + (1.41) + (10.43) + (19.3) + (28.12) = 3927$

$R_1.R_7 = (30.22) + (39.31) + (48.40) + (1.49) + (10.2) + (19.11) + (28.20) = 4627$

$R_2.R_3 = (38.6) + (47.6) + (7.8) + (9.17) + (18.26) + (27.35) + (29.37) = 4725$

$R_2.R_4 = (38.5) + (47.14) + (7.16) + (9.25) + (18.34) + (27.36) + (29.45) = 4074$

$R_2.R_5 = (38.13) + (47.15) + (7.24) + (9.33) + (18.42) + (27.44) + (29.4) = 3724$

$R_2.R_6 = (38.21) + (47.23) + (7.32) + (9.41) + (18.43) + (27.3) + (29.12) = 3675$

$R_2.R_7 = (38.22) + (47.31) + (7.40) + (9.49) + (18.2) + (27.11) + (29.20) = 3927$

$R_3.R_4 = (46.5) + (6.41) + (8.16) + (17.25) + (26.34) + (35.36) + (37.45) = 4676$

$R_3.R_5 = (46.13) + (6.15) + (8.24) + (17.33) + (26.42) + (35.44) + (3.74) = 4221$

$R_3.R_6 = (46.21) + (6.23) + (8.32) + (17.4) + (26.43) + (35.3) + (37.12) = 3724$

$R_3.R_7 = (46.22) + (6.31) + (8.40) + (17.49) + (26.2) + (35.11) + (37.20) = 3528$

$R_4.R_5 = (5.13) + (14.15) + (16.24) + (25.33) + (34.42) + (36.44) + (45.4) = 4676$

$R_4.R_6 = (5.21) + (14.23) + (16.34) + (25.41) + (34.43) + (36.3) + (45.12) = 4074$

$R_4.R_7 = (5.22) + (14.31) + (16.40) + (25.49) + (34.2) + (36.11) + (45.20) = 3773$

$R_5.R_6 = (13.21) + (15.23) + (24.32) + (33.41) + (42.43) + (44.3) + (4.12) = 4725$

$R_5.R_7 = (13.22) + (15.31) + (24.40) + (33.49) + (42.2) + (44.11) + (4.20) = 3976$

$R_6.R_7 = (21.22) + (23.31) + (32.40) + (41.49) + (43.2) + (3.11) + (12.20) = 4823$

To find the dot product of columns from Table 1.7 above, we compute:

$C_1.C_2 = (30.39) + (48.47) + (46.6) + (5.14) + (13.15) + (21.23) + (22.31) = 4662$

$C_1.C_3 = (30.48) + (48.7) + (46.8) + (5.16) + (13.24) + (21.32) + (22.40) = 4018$

$C_1.C_4 = (30.1) + (48.9) + (46.17) + (5.25) + (13.33) + (21.41) + (22.49) = 3647$

$C_1.C_5 = (30.10) + (48.18) + (46.24) + (5.34) + (13.42) + (21.43) + (22.2) = 3843$

$C_1.C_6 = (30.19) + (48.27) + (46.35) + (5.36) + (13.44) + (21.3) + (22.11) = 4263$

$C_1.C_7 = (30.28) + (48.29) + (46.37) + (5.45) + (13.4) + (21.12) + (22.20) = 4613$

$C_2.C_3 = (3.46) + (47.7) + (6.8) + (14.18) + (15.24) + (23.32) + (31.40) = 4809$

$C_2.C_4 = (39.1) + (47.9) + (6.17) + (14.25) + (15.33) + (23.41) + (31.49) = 3871$

$C_2.C_5 = (39.10) + (47.18) + (6.26) + (14.34) + (15.42) + (23.43) + (31.2) = 3549$

$C_2.C_6 = (39.19) + (47.27) + (6.35) + (14.36) + (15.44) + (23.6) + (31.4) = 3794$

$C_2.C_7 = (39.28) + (47.29) + (6.37) + (14.45) + (15.4) + (23.12) + (31.20) = 4263$

$C_3.C_4 = (48.1) + (7.9) + (8.17) + (16.25) + (24.33) + (32.41) + (40.49) = 4711$

$C_3.C_5 = (48.10) + (7.18) + (8.26) + (16.34) + (24.42) + (32.43) + (40.2) = 3822$

$C_3.C_6 = (48.19) + (7.27) + (8.35) + (16.36) + (24.44) + (32.3) + (40.11) = 3549$

$C_3.C_7 = (48.28) + (7.29) + (8.37) + (16.45) + (24.4) + (32.12) + (40.20) = 3843$

$C_4.C_5 = (1.10) + (9.18) + (17.26) + (25.34) + (33.42) + (41.43) + (49.2) = 4711$

$C_4.C_6 = (1.19) + (9.27) + (17.35) + (25.36) + (33.44) + (41.3) + (49.11) = 3871$

$C_4.C_7 = (1.28) + (9.29) + (17.37) + (25.45) + (33.4) + (41.12) + (49.20) = 3647$

$C_5.C_6 = (10.19) + (18.27) + (26.35) + (34.36) + (42.44) + (43.3) + (2.11) = 4809$

$C_5.C_7 = (10.28) + (18.29) + (26.37) + (34.45) + (42.4) + (43.12) + (2.20) = 4018$

$C_6.C_7 = (19.28) + (27.29) + (35.37) + (36.45) + (44.4) + (3.12) + (2.20) = 4662$

From the dot row product obtained above, the following are the matching pairs:

$R_1.R_2 = R_6.R_7$, $R_1.R_3 = R_5.R_7$, $R_1.R_4 = R_4.R_7$, $R_1.R_5 = R_3.R_7$, $R_1.R_6 = R_2.R_7$, $R_2.R_3 = R_5.R_6$, $R_2.R_4 = R_4.R_6$, $R_2.R_5 = R_3.R_6$, $R_3.R_4 = R_4.R_5$.

Also from the dot column product obtained above, the following are the matching pairs

$C_1.C_2 = C_6.C_7$, $C_1.C_3 = C_5.C_7$, $C_1.C_4 = C_4.C_7$, $C_1.C_5 = C_3.C_7$, $C_1.C_6 = C_2.C_7$, $C_2.C_3 = C_5.C_6$, $C_2.C_4 = C_4.C_6$, $C_2.C_5 = C_3.C_6$, $C_3.C_4 = C_4.C_5$.

Construction of the dot product of De Mezeriac magic square of order 7x7 above:

Using the formula $R_p.R_q = \sum_{i=1}^{n} a_{pi}.a_{qi}$,

the dot product of rows are computed from table 2.7 above as:

$R_1.R_2 = (4.35) + (29.11) + (12.36) + (37.19) + (20.44) + (45.27) + (28.3) = 3773$

$R_1.R_3 = (4.10) + (29.42) + (12.18) + (37.43) + (20.26) + (45.2) + (28.34) = 4627$

$R_1.R_4 = (4.41) + (29.17) + (12.49) + (37.25) + (20.1) + (45.33) + (28.9) = 3927$

$R_1.R_5 = (4.16) + (29.48) + (12.24) + (37.7) + (20.32) + (45.8) + (28.40) = 4123$

$R_1.R_6 = (4.47) + (29.23) + (12.6) + (37.31) + (20.14) + (45.39) + (28.15) = 4529$

$R_1.R_7 = (4.22) + (29.5) + (12.30) + (37.13) + (20.38) + (45.21) + (28.46) = 4863$

$R_2.R_3 = (35.10) + (11.42) + (36.18) + (19.43) + (44.26) + (27.2) + (3.34) = 3577$

$R_2.R_4 = (35.41) + (11.17) + (36.49) + (19.25) + (44.1) + (27.33) + (3.9) = 4823$

$R_2.R_5 = (35.16) + (11.48) + (36.24) + (19.7) + (44.32) + (27.8) + (3.40) = 3829$

$R_2.R_6 = (35.47) + (11.23) + (36.6) + (19.31) + (44.14) + (27.37) + (3.15) = 4417$

$R_2.R_7 = (35.22) + (11.5) + (36.30) + (19.13) + (44.38) + (27.21) + (3.46) = 4529$

$R_3.R_4 = (10.44) + (42.17) + (18.49) + (43.25) + (26.1) + (2.33) + (34.9) = 3479$

$R_3.R_5 = (10.16) + (42.48) + (18.24) + (43.7) + (26.32) + (2.8) + (34.40) = 5117$

$R_3.R_6 = (10.47) + (42.23) + (18.6) + (43.31) + (26.14) + (2.39) + (34.15) = 3829$

$R_3.R_7 = (10.22) + (42.5) + (18.30) + (43.13) + (26.38) + (2.21) + (34.46) = 4123$

$$R_4.R_5 = (41.16) + (17.48) + (49.24) + (25.7) + (1.32) + (33.8) + (9.40) = 3479$$

$$R_4.R_6 = (41.47) + (17.23) + (49.6) + (25.31) + (1.14) + (33.39) + (9.15) = 4823$$

$$R_4.R_7 = (41.22) + (17.5) + (49.30) + (25.13) + (1.38) + (33.21) + (9.46) = 3927$$

$$R_5.R_6 = (16.47) + (48.23) + (24.6) + (7.31) + (32.14) + (8.39) + (40.15) = 3577$$

$$R_5.R_7 = (16.22) + (48.5) + (24.30) + (7.13) + (32.38) + (8.21) + (40.46) = 4627$$

$$R_6.R_7 = (47.22) + (23.5) + (6.30) + (31.13) + (14.33) + (34.21) + (15.46) = 3773$$

To find the dot product of columns from Table 2.7 above, we compute:

$$C_1.C_2 = (4.29) + (35.11) + (10.42) + (41.17) + (16.48) + (7.23) + (22.5) = 3577$$

$$C_1.C_3 = (4.12) + (35.36) + (10.18) + (41.49) + (16.24) + (7.6) + (22.30) = 4823$$

$$C_1.C_4 = (4.37) + (35.19) + (10.43) + (41.25) + (16.7) + (7.31) + (22.13) = 4123$$

$$C_1.C_5 = (4.20) + (35.44) + (10.26) + (41.1) + (16.32) + (7.14) + (22.28) = 3927$$

$$C_1.C_6 = (4.45) + (35.27) + (10.2) + (41.33) + (16.8) + (7.39) + (22.21) = 4921$$

$$C_1.C_7 = (4.28) + (35.3) + (10.34) + (41.9) + (16.40) + (7.15) + (22.46) = 3283$$

$$C_2.C_3 = (29.12) + (11.36) + (42.18) + (17.49) + (48.24) + (23.6) + (5.30) = 3773$$

$$C_2.C_4 = (29.37) + (11.19) + (42.43) + (17.25) + (48.7) + (23.31) + (5.13) = 4627$$

$$C_2.C_5 = (29.20) + (11.44) + (42.26) + (17.1) + (48.32) + (23.14) + (5.58) = 4221$$

$$C_2.C_6 = (29.45) + (11.27) + (42.2) + (17.33) + (48.8) + (23.39) + (5.21) = 3633$$

$$C_2.C_7 = (29.28) + (11.3) + (42.34) + (17.9) + (48.40) + (23.15) + (5.46) = 4921$$

$$C_3.C_4 = (12.37) + (36.19) + (18.43) + (49.25) + (24.7) + (6.31) + (30.13) = 3871$$

$$C_3.C_5 = (12.20) + (36.44) + (18.26) + (49.1) + (24.32) + (6.14) + (30.38) = 4333$$

$$C_3.C_6 = (12.45) + (36.27) + (18.2) + (49.33) + (24.8) + (6.39) + (30.21) = 4221$$

$$C_3.C_7 = (12.28) + (36.3) + (18.34) + (49.9) + (24.40) + (6.15) + (30.46) = 3921$$

$$C_4.C_5 = (37.20) + (91.44) + (43.26) + (25.1) + (7.32) + (31.14) + (13.38) = 3871$$

$$C_4.C_6 = (37.45) + (91.27) + (43.2) + (25.33) + (7.8) + (31.39) + (13.21) = 4627$$

$$C_4.C_7 = (37.28) + (91.3) + (43.34) + (25.9) + (7.40) + (31.15) + (13.46) = 4123$$

$$C_5.C_6 = (20.45) + (44.27) + (26.2) + (1.38) + (32.8) + (14.39) + (38.21) = 3773$$

$$C_5.C_7 = (20.28) + (44.3) + (26.34) + (1.9) + (32.40) + (14.15) + (38.46) = 4823$$

$$C_6.C_7 = (45.28) + (27.3) + (2.34) + (33.9) + (8.40) + (39.15) + (21.46) = 3577$$

From the dot row product obtained above, the following are the matching pairs:

$R_1.R_2 = R_6.R_7$, $R_1.R_3 = R_5.R_7$,
$R_1.R_4 = R_4.R_7$, $R_1.R_5 = R_3.R_7$,
$R_1.R_6 = R_2.R_7$,
$R_2.R_3 = R_5.R_6$, $R_2.R_4 = R_4.R_6$,
$R_2.R_5 = R_3.R_6$, $R_3.R_4 = R_4.R_5$.

Also from the dot column product obtained above, the following are the matching pairs:

$C_1.C_2 = C_6.C_7$, $C_1.C_3 = C_5.C_7$,
$C_1.C_4 = C_4.C_7$, $C_1.C_5 = C_3.C_7$,
$C_1.C_6 = C_2.C_7$,
$C_2.C_3 = C_5.C_6$, $C_2.C_4 = C_4.C_6$,
$C_2.C_5 = C_3.C_6$, $C_3.C_4 = C_4.C_5$.

4.0 CONCLUSION

The same pairs of row dot products match for column dot product in both Loubere and Mezeriac methods, although the sum of the matching pair row dot product and the column dot product are not the same. It is interesting to note that any dot product of row or column whose sum of subscript equals 8 does not have matching pair. Otherwise, any dot product of row or column whose sum of subscript is greater or less than 8 has a matching pair.

From the construction of row and column dot product of both Loubere and Mezeriac magic squares, it is seen that both methods have the same pattern of matching pairs of both the row and column dot product for magic square of order seven, but the Loubere method is easier to construct and more general compared to Mezeriac method.

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