PLANE STAGNATION DOUBLE-DIFFUSIVE MHD CONVECTIVE FLOW UNDER THE PRESENCE OF VISCOUS DISSIPATION, THERMAL RADIATION AND UNIFORM MAGNETIC FIELD

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ABSTRACT

In this paper, plane stagnation double-diffusive MHD convective flow with convective boundary conditions in the presence of thermal radiation and uniform magnetic field is investigated. The governing non-linear partial differential equations have been reduced to a system of nonlinear coupled ordinary differential equations with the use of similarity transformations. The resulting equations were solved numerically using the classical fourth order Runge-Kutta formula together with shooting technique implemented on a computer program. The effects of the physical parameters were observed on the velocity, temperature and concentration profiles. Computational analyses for the skin-friction coefficients, Nusselt and Sherwood numbers were made and presented through tables and graphical plots for various fluid parameters.

Keywords: Plane Stagnation, Double-Diffusive, MHD Convective Flow, Thermal Radiation, Uniform Magnetic Field

1.0 INTRODUCTION

Studies abound on the stagnation point flow in literatures. Over the years, the flow near a stagnation point has caught the attention of many researchers due to its numerous scientific and industrial applications. Knowledge of the flow near a stagnation point has been found applicable and extremely useful in industrial processes like nuclear reactors, extraction of polymers, cooling of electronics devices, (Crane, 1970) researched on the two dimensional steady flow of an incompressible viscous fluid caused by a linearly stretching plate. Laminar mixed convection in two dimensional stagnation flows around heated surfaces was considered by (Ramachand, 1988). He investigated cases of an arbitrary wall temperature and arbitrary heat flux variation. The results of the investigation showed that for a specified range of buoyancy parameter dual solutions existed and a reversed flow developed in the buoyancy opposing flow region. In their work, (Sharma and Singh, 2009) analysed the effects of variable thermal conductivity and heat source/sink on MHD flow near stagnation point on linearly stretching sheet. They inferred that the rate of heat transfer at the sheet increases due to increase in the thermal conductivity parameter, however it decreases due to increases in the ratio of free stream velocity parameter to stretching sheet parameter, in absence of magnetic field and volumetric rate of heat/sink parameter.

Of concern in this research are the effects of thermal radiation in industrial processes. It has been established that thermal radiation effects plays an important role in scientific processes such as cooling of a metal or glass sheet. In the light of these applications, (Samad and Rahman, 2006) investigated the thermal radiation interaction on an absorbing emitting fluid past a vertical porous plate immersed in a porous

medium. Steady radiated free convective flow along a vertical flat plate in the presence of magnetic field was investigated by (Emmanuel and Uddin, 2011). Their result showed that magnetic field can control the heat transfer and radiation has a significant effect on the velocity as well as temperature distributions.

Convective boundary condition is used mostly to describe a linear convective heat exchange condition for one or more algebraic entities in thermal. Thermal energy storage, nuclear plants, gas turbines, are processes that define heat transfer analysis with convective boundary conditions. On the flow with convective boundary conditions, (Aziz, 2009) studied a similarity solution applied to laminar thermal boundary layer flow over flat plate with convective surface boundary conditions but only obtained local Biot number which was made global on restricted conditions. Not long ago, (Okedayo et al., 2011) examined the effects of viscous dissipation on the mixed convection heat transfer over a flat plate with internal heat generation and convective boundary condition. Also, (Okedayo et al., 2012a) obtained the similarity solution to the plane stagnation point flow with convective boundary conditions.

(Okedayo et al., 2012b) again examined plane stagnation double-diffusive MHD convective flow with convective boundary condition in a porous media. A numerical solution of the problem was obtained using the classical Runge-Kutta method together with shooting technique. Thereafter, (Adeola and Adekunle, 2014) did an extension of the work of (Okedayo et al., 2012b) by included thermal radiation and viscous dissipation into their models. They both investigated effects of some thermo-physical properties on force convective stagnation point on a stretching sheet with convective boundary condition in the presence of thermal radiation and magnetic field.

Recently, (Bognar, 2016) presented a numerical method for the boundary layer problem of non-

Newtonian fluid flow along moving surfaces using iterative transformation method, applying similarity transformation to the governing partial differential equations. The work exhibited the drag co-efficient dependence on the velocity ratio and on the power-law exponent. Thereafter, (Fenuga et al., 2018) carried out analysis of thermal boundary layer flow over a vertical plate with electrical conductivity and convective surface boundary conditions using Runge-Kutta fourth order method with shooting technique. The behaviour and properties of thermo-physical parameters in the fluid flow structure of the velocity and temperature fields were examined.

In the presence paper, force convective stagnation point on a stretching sheet with convective boundary conditions in the presence of thermal radiation and magnetic fields is investigated. The inclusion of thermal radiation and viscous dissipation extended the work of (Bognar, 2016); (Fenuga et al., 2018). In addition, the effects of various parameters on the velocity, temperature and concentration profiles were presented.

2.0 MATHEMATICAL FORMULATION

A steady two-dimensional MHD flow of a viscous, incompressible and electrically conducting fluid of temperature T_{∞} along a heated vertical flat plate is considered. The flow is assumed to be in x-direction, which is chosen along the plate in the upward direction and y-axis normal to the plate. The geometric of the flow is presented in figure one. It is assumed that the free stream velocity is of the form $(x_0)x_0$ where x_0 where x_0 where x_0 where x_0 where x_0 constant.

The fluid is considered to be a gray, absorbing emitting radiation but non-scattering medium and Roseland approximation is used to describe the radiative heat flux in the energy equation. The radiative heat flux in the x-direction is negligible to the flux in the y-direction. Further, a uniform magnetic field of strength oB is assumed to be

applied in the positive y-direction normal to the flat plate. The magnetic Reynolds number is assumed to small, and thus the induced magnetic field is negligible. The plate temperature is initially raised to T_w (where $T_f > T_w > T_\infty$).

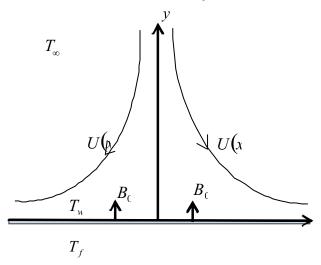


Figure 2.1: Physical model and coordinate system of the problem

Under the above assumptions and taking the usual Boussinesq's approximation into account, the governing equations relevant for the model, namely the continuity, momentum, energy and concentration are presented below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \gamma\frac{\partial^{2} u}{\partial y^{2}} + S(x)g\beta(T - T(\infty))$$

$$+ S(x)g\beta(C - C(\infty)) - \frac{\sigma B_{0}^{2} u}{\rho} - \frac{\gamma}{k}u$$
(2)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial T}{\partial y} + \frac{\gamma}{c_p} \left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y}$$
 (3)

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}$$
 (4)

The corresponding boundary conditions are:

$$u = 0 \quad v = 0 \quad \frac{-k\partial T(x,0)}{\partial y} = h_f \left(T_f - T(x,0)\right)$$
$$C = C_w - C_\infty \text{ at } y = 0$$
 (5a)

$$u = U(x)$$
 $T \to T_{\infty}$ $C \to C_{\infty}$ (5b)
As $v \to \infty$

By using Roseland approximation q takes the form

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial v} \tag{6}$$

Where * σ is the Stefan-Boltman constant and *kis the mean absorption coefficient. It is assumed that the temperature differences within the flow are sufficiently small such that ${}_4T$ may be expressed as a linear function of temperature. This is accomplished by expanding ${}_4T$ in a Taylor series about ${}_\infty T$ and neglecting higher-order terms, thus

$$T^{4} \approx T_{\infty}^{4} + (T - T_{\infty}) 4T_{\infty}^{3} = T_{\infty}^{4} + 4TT_{\infty}^{3}$$
$$-4T_{\infty}^{4} = 4TT_{\infty}^{3} - 3T_{\infty}^{4}$$
(7)

Put equation (7) into equation (6). We have

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} = -\frac{4\sigma^*}{3k^*} \frac{\partial}{\partial y} \left(4TT_{\infty}^3 - 3T_{\infty}^4 \right)$$
$$= -\frac{16\sigma^* T_{\infty}^3}{3k^*} \frac{\partial T}{\partial y}$$

In order to solve the equations (1)-(5), we introduce the following similarity variables and dimensionless numbers:

$$\eta = y \sqrt{\frac{U(x)}{\gamma x}}, \ \psi(x, y) = \sqrt{\gamma x U(x)} f(\eta),$$
 (9a)
$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}$$
 (9b)
$$P_r = \frac{\gamma}{\alpha}, M = \frac{\sigma B_0^2}{\rho U_0}, \quad \theta = \frac{T - T_\infty}{T_f - T_\infty},$$
 (9c)

$$\varphi = \frac{C - C_{\infty}}{C_{w} - C_{\infty}},\tag{9d}$$

$$S_c = \frac{\gamma}{D}, G_r = \frac{S(x)g\beta(T_{\infty} - T_{w})}{U_0^2 x}, \quad (9e)$$

$$G_{m} = \frac{S(x)g\beta \left(C_{\infty} - C_{w}\right)}{U_{0}^{2}x} \tag{9f}$$

$$U(x) = U_0 x, \quad S(x) = x \tag{9g}$$

$$E_{c} = \frac{U_{0}^{2} x^{2}}{c_{p} (T_{w} - T_{\infty})}, \ N = \frac{k k^{*}}{4 \sigma^{*} T_{\infty}^{2}}$$
(9h)

We apply equations (9b) on equation (1) to obtain:

$$\frac{\partial^2 \Psi}{\partial x \partial y} - \frac{\partial^2 \Psi}{\partial x \partial y} = 0 \tag{10}$$

Hence, equation (1) is satisfied. Equally, we apply equations (9a, 9c and 9d) on equation (2), since the flow is inviscid (Okedayo et al., 2012a), we have:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U(x)\frac{dU(x)}{dx} + \gamma \frac{\partial^{2} u}{\partial y^{2}} + S(x)g\beta(T - T(\infty)) + S(x)g\beta(C - C(\infty))$$
$$-\frac{\sigma B_{0}^{2}}{\rho}(u - U(x)) - \frac{\gamma}{k}(u - U(x))$$
(11)

Applying equations (9a-9h) on equation (11), we have as follows:

$$u = \frac{\partial \Psi}{\partial y} = \frac{\partial}{\partial y} \left(\sqrt{\gamma x U(x)} f \right) = \sqrt{\gamma x U(x)} \frac{\partial}{\partial \eta} f \frac{\partial \eta}{\partial y}$$

$$= \sqrt{\gamma x U(x)} f' \frac{\partial}{\partial y} \left(y \sqrt{\frac{U(x)}{\gamma x}} \right) = \sqrt{\gamma x U(x)} f' \sqrt{\frac{U(x)}{\gamma x}}$$

$$= U(x) f' = U_0 x f'$$
(12)

$$v = -\frac{\partial \Psi}{\partial x} = -\frac{\partial}{\partial x} \left(\sqrt{\gamma x U(x)} f \right) = -\frac{\partial}{\partial x} \left(\sqrt{\gamma x^2 U_0} f \right)$$
$$= -f \sqrt{\gamma U_0} \frac{\partial}{\partial x} (x) = -\sqrt{\gamma U_0} f$$
(13)

Thus

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \left(U_0 x f' \right) = U_0 f' \tag{14}$$

Similarly

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y} \left(U_0 x f' \right) = \frac{\partial}{\partial \eta} \left(U_0 x f' \right) \frac{\partial \eta}{\partial y}$$

$$= U_0 x f'' \frac{\partial}{\partial y} \left(y \sqrt{\frac{U(x)}{\gamma x}} \right) = U_0 x f'' \frac{\partial}{\partial y} \left(y \sqrt{\frac{U_0 x}{\gamma x}} \right)$$

$$= U_0 x f'' \sqrt{\frac{U_0}{\gamma}}$$
(15)

Also

$$\frac{\partial^{2} u}{\partial y^{2}} = \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \right) = \frac{\partial}{\partial y} \left(U_{0} x f^{\dagger} \sqrt{\frac{U_{0}}{\gamma}} \right) \\
= \frac{\partial}{\partial \eta} \left(U_{0} x f^{"} \sqrt{\frac{U_{0}}{\gamma}} \right) \frac{\partial \eta}{\partial y} = U_{0} x f^{"} \sqrt{\frac{U_{0}}{\gamma}} \frac{\partial}{\partial y} \left(y \sqrt{\frac{U(x)}{\gamma x}} \right) \\
= U_{0} x f^{"} \sqrt{\frac{U_{0}}{\gamma}} \sqrt{\frac{U_{0}}{\gamma}} = \frac{U_{0}^{2} x f^{"}}{\gamma}$$
(16)

Substitute equations (12)-(16) into equation (11), expanded, divided by xU_{20} and re-arranged, we have

$$f''' + ff'' - f'^{2} + G_{T}\theta + G_{m}\phi - \left(H + \frac{1}{D_{a}}\right)(f' - 1) + 1 = 0$$
(17)

Similarly, we transformed the energy equation (3) and equation (4), using equation (9) which yields respectively:

$$(3N+4)\theta' + 3NP_r f\theta' + 3NE_c P_r f^{*2} = 0$$
 (18)

and

$$\varphi'' + S_c f \varphi' = 0 \tag{19}$$

Hence, the dimensionless governing equations to be solved together with their transformed boundary conditions are

$$f''' + ff'' - f'^{2} + G_{T}\theta + G_{m}\phi$$

$$-\left(H + \frac{1}{D_{a}}\right)(f' - 1) + 1 = 0$$
(20)

$$(3N+4)\theta'+3NP_r f\theta'+3NE_c P_r f''^2=0$$
 (21)

$$\varphi'' + S_c f \varphi' = 0 \tag{22}$$

$$f'(0) = f(0); \quad \theta'(0) = B_i(1 - \theta(0));$$

$$\varphi(0) = \theta(\infty) = \varphi(\infty) = 0; \quad f(\infty) = 1$$
(23)

Where the prime denotes differentiation with respect to η . The existence and uniqueness the solutions of the dimensionless governing equations (20-22) was established using the approach in (Olanrewaju et al., 2007).

3.0 SOLUTION OF THE PROBLEM AND FINDINGS

Equations (20)-(22) together with their boundary conditions in equation (23) are coupled nonlinear ordinary differential equations which are difficult to solve by known available analytical methods. Thus, in order to solve the governing equations, we seek a numerical solution by employing the classical fourth order Runge-Kutta method and shooting technique used in (Olanrewaju et al 2007), implemented on a computer program written in Matlab.. A convenient step size was chosen to obtain the desired accuracy.

The effects of various parameters on the velocity profile, temperature profile and concentration profile were computed and presented in figure 1-9 and followed by discussion. The fluid parameters were assigned the following values;

$$G_t = 0.1, G_m = 0.1, E_c = 0.05, P_r = 0.71, H = 0.1,$$

 $N = 1, Bi = 0.2, D_a = 0.1$

and 2.0=cSrespectively except where stated otherwise. Figures 1 -4 depict the velocity distribution, highlighting the effects of HGGmt, and aDrespectively. It is observed an increase in thermal Grashof number G, solute G r a s h o f n u m b e r m G o r m a g n e t i c parameter Hincreases the velocity profile. However, increase in Darcy number leads to a decrease in velocity distribution. Furthermore, it

is observed that the velocity starts minimum value of zero at the plate surface and increases exponentially to a free stream value of unity away far from the surface, satisfying the field boundary conditions for all parameter values.

Figures 5-8 show the temperature distribution for various values of NEBic,, and rPrespectively. Figures 5 and 6 reveal that there is an increase in the thermal boundary layer and a rise in fluid temperature as the values of Biot Bior Eckert cEnumber is increased. Further, figures (7) and (8) show the effects of Prandtl number rPand radiation parameter on the temperature profile. An increase in thermal boundary layer and a rise in fluid temperature are observed only near boundary while a decrease in temperature is observed for the two parameters as we move away from the boundary. Furthermore, it is observed that the maximum values for the temperature are attained at the boundary and then decreases to zero as we move away from the surface, satisfying the field boundary conditions for all the parameter values.

The effects of the Schmidt number Son the species concentration are presented in figure 9. The concentration profile has a maximum value at the plate surface and decreases exponentially to the free stream zero value far from the plate. It is observed that an increase in the Schmidt number decreases the concentration of the fluid.

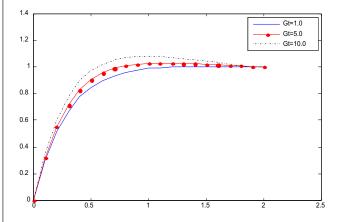


Figure 1: Velocity profile for various values of thermal Grashof number

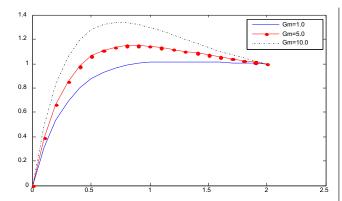


Figure 2: Velocity profile for various values of solute Grashof number

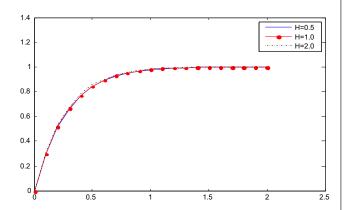


Figure 3: Velocity profile for various values of magnetic parameter

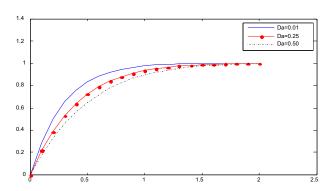


Figure 4: Velocity profile for various values of Darcy number

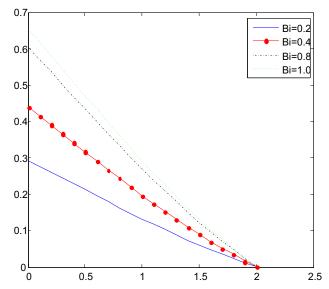


Figure 5: Temperature profile for various values of Biot number

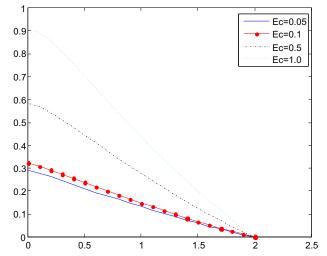


Figure 6: Temperature profile for various values of Eckert number

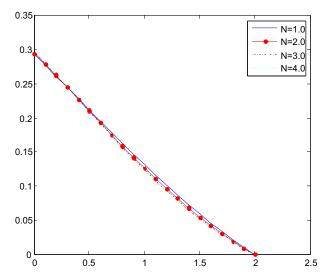


Figure 7: Temperature profile for various values of Radiation number

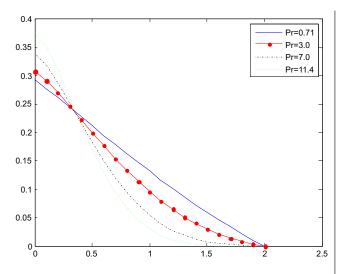


Figure 8: Temperature profile for various values of Prandtl number

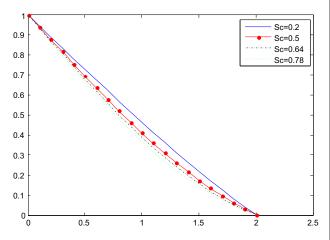


Figure 9: Concentration profile for various values of Schemidt number

4.0. DISCUSSION AND CONCLUSION

In this paper, steady two dimensional plane stagnation point flow of a viscous incompressible and electrically conducting fluid towards heated vertical flat plate was considered under most reasonable physical assumptions. The fluid is considered to be a gray, absorbing emitting radiation but non-scattering medium and Roseland approximation was used to describe the radiative heat flux in the energy equation. The constitutive governing equations for our model were reduced to a system of nonlinear coupled ordinary differential equations with the use of similarity transformations. The existence and uniqueness of the governing equations were

proved and formerly established. The mathematical models describing flows under consideration are in the form of complex coupled differential equations for which solutions are not easy to obtain using analytical methods. Thus, numerical technique has been utilized by using the classical fourth order Runge-Kutta formula together with shooting technique implemented on a computer program. The solutions for various values of fluid parameters such thermal Grashof number, solute Grashof number, Prandtl number, Radiation parameter, Biot number, number, Schmidt number ,Eckert number and magnetic parameters were obtained. Moreover, Computational analyses for the skin-friction coefficients, Nusselt and Sherwood number were made and presented through tables for various fluid parameters. Graphical representation were also drawn and discussed.

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