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ON HEAT TRANSFER ANALYSIS FOR SQUEEZING MAGNETO HYDRODYNAMIC FLOW BETWEEN PARALLEL PERMEABLE DISKS USING ADOMIAN DECOMPOSITION METHOD

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ABSTRACT
In this paper, heat transfer analysis for the squeezing magneto-hydrodynamic MHD flow of a viscous incompressible fluid between disks is considered. Upper disk is movable in upward and downward directions while the lower disk is fixed but permeable. Viable similarity transforms are used to convert the conservation law equations to a system of nonlinear ordinary differential equations. The resulting system is solved by using Adomian Decomposition Method (ADM).

It was observed that the behaviour of all physical parameters is opposite on velocity profile in the cases of suction and injection. On the other hand, the effect of parameters remain similar on temperature profile for both suction and injection cases.

Keywords: *Squeezing, magneto-hydrodynamic, Magnetic field, Adomian Decomposing Method ADM, Parallel disks*

1.0 INTRODUCTION
Squeezing flow between the parallel disks has received the attention of recent researchers due to widespread applications of such flows in various mechanical engineering disciplines. The flow is generated because of two parallel approaching surfaces in relative motion. The parallel approaching surfaces phenomena along with the relative motion is mostly used by the engineers in the modeling of flow of oil in bearings, determination of capacity of load-bearings, compression and injection modeling. (Hussain, et al, 2012); (Stefan ,1874) reported the squeezing flow for lubrication approximation. (Domairry and Aziz,2009) studied magnetohydrodynamic squeezing flow of viscous liquid bounded by parallel disks. (Siddiqui et al.,2008) examined squeezing flow subject to an applied magnetic field. (Rashidi et al., 2010) performed an analysis of hydrodynamic squeezing flow by developing series solutions. Some other investigations on squeezing flow can be seen in the studies (Qayyum et al., 2010). (Khan et al., 2014) employed variation iteration method. Furthermore, another area of interesting in the study of squeezing flow in the past was directed to the peristalsis of Newtonian and non-Newtonian liquids in a straight channel which seems not realistic in several applications relevant to physiological and engineering processes , (Ali et al., 2010);(Hayat et al., 2012); (Tripathi , 2012); (Mekheimer et al., 2013); (Vairavelua et al., 2013); (Mustapa et al., 2014); (Shehzad et al., 2015); (Hayat et al., 2017); and (Hayat et al., 2018). In this paper, effort is made to establish the results of the aforementioned Researchers, most especially, (Domairy and Aziz, 2009), Hussain et al., 2012) and (Hayat et al., 2018) using Adomian Decomposition Method (ADM).

2.0 MATHEMATICAL FORMULATIONS
MHD flow of vicious incomprehensible fluid is taken into consideration through a system consisting of two parallel disks distance $h(t) = H(1 - at)^{½}$ apart. Magnetic field proportional to $B_0(1 - at)^{½}$

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is applied normal to the disks. It is assumed that there is no induced magnetic field. T_w and T_h represent the constant temperature at $Z=0$ and $Z=h$ (t) respectively. Upper disk at $Z=h$ (t) is moving with velocity $\frac{aH(1-at)^{\frac{1}{2}}}{2}$ towards or away from the static lower but permeable disk at $z = 0$ We choose the cylindrical coordinate system $(r,\ \emptyset,\ z)$ Rotational symmetry flow $\left(\frac{\partial}{\partial\emptyset} = 0\right)$ allows us to take the azimuthal component of v of the velocity $V_{=}$(u, v, w) equal to zero. As a result, the governing equation for unsteady two dimensional flow and heat transfer of a viscous fluid can be written as

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \qquad (1)$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = \frac{-\partial p}{\partial r} + \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) - \frac{\sigma}{\rho}B^2(t) \qquad (2)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial u}{\partial z}\right) = \frac{-\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{\partial^2 w}{\partial z^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) \qquad (3)$$

$$C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial u}{\partial t}\right) = \frac{K_0}{\rho}\left(\frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial T}{\partial r} - \frac{u}{r^2}\right) + v\left\{2\frac{u^2}{r^2} + \left(\frac{\partial u}{\partial z}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + 2\left(\frac{\partial u}{\partial r}\right)^2 + 2\left(\frac{\partial w}{\partial r}\right)^2 + 2\frac{\partial u}{\partial z}\frac{\partial w}{\partial r}\right\} \qquad (4)$$

Auxiliary Conditions

$$u = 0, \quad w = \frac{\partial h}{\partial t} \quad \text{at } z = h(t) \qquad (5)$$

$$u = 0, \quad w = -w_0 \quad at\ z = 0.$$

$$T = T_w, \quad at\ z = 0.$$

$$u = T_h \quad at\ z = h(t). \qquad (6)$$

u and w here are the velocity components in r and z directions respectively, μ is dynamic viscosity, p is the pressure and p is density. Further T denotes temperature, K_o thermal conductivity, C_p is specific heat, v is kinematic viscosity and W_0 is suction/ injection velocity.

Using the transformation (5)

$$u = \frac{ar}{2(1-at)}f(\eta), \qquad = -\frac{aH}{\sqrt{1-at}}f(\eta),$$

$$B(t) = \frac{B_0}{\sqrt{1-at}}, \eta = \frac{z}{H\sqrt{1-at}},$$

$$\theta = \frac{T - T_h}{T_w - T_h},$$

Equation (2) – (4) and eliminating pressure terms from the resulting equation, we obtain

$$f^{1111} - S(\eta f^{111} + 3f^{11} - 2ff^{111}) - M^2 f^{11} = 0 \qquad (8)$$

$$\theta^{11} - S\,Pr(2f\theta^1 - \eta\theta^1) - \Pr\,Ec(f^{112} + 12\delta^2 f^{12}) \qquad (9)$$

With the associated conditions

$$f(0) = A, \quad f^1(0) = 0, \quad \theta(0) = 1,$$

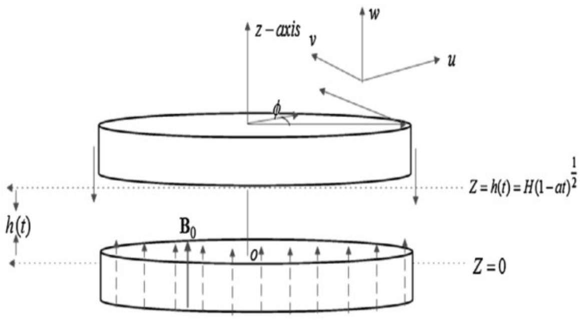
$$f(1) = \tfrac{1}{2}, \quad f^1(1) = 0, \quad \theta(0) = 1$$

Where S denotes the squeeze number, A is Suction/injection parameter, M is Hartman number, Pr Prandtl number, Ec Modified Eckert number, and δ denotes the dimensionless length defined as

$$S = \frac{aH^2}{2v}, \quad M^2 = \frac{B_0^2 H^2}{v}, \quad \Pr\frac{\mu C_p}{K_0},$$

$$Ec = \frac{aH^2}{C_p(T_w - T_h)}\left(\frac{ar}{2(1-at)}\right)^2, \quad \delta^2 = \frac{H^2(1-at)}{r^2}$$

Skin friction co-efficient and Nusselt number are defined in terms of variable 7 as



Geometry of the problem

$$f^{11}(1) = \frac{H^2}{r^2} Re_r C_{fr}, (1-at)^{½} N = -\theta^1(1) \quad (12)$$

$$Re_r = \frac{raH(1-at)^{½}}{2v} \quad (13)$$

3.0 Adomian Decomposition Method (ADM) Solution Procedure

$$\varphi[u(y)] = g(y) \quad (14)$$

Where φ represents a functional differential operator, then equation (14) is decomposed into the form $L + R$ where L is taken as invertible highest order and R is the linear differential operator of order less than L.

Therefore, equation (14) can be expressed as

$$LU + RU + NU = g(y) \quad (15)$$

$$u = L^{-1}g - L^{-1}(Ru - Nu) \quad (1)$$

In this case, L is the forth order operator, then L^{-1} is a four fold integral.

Now equation (16) takes the form

$$u = \sum_{j=0}^{3} aj\frac{yj}{j!} + L^{-1}g - L^{-1}(RU - NU) \quad (17)$$

Where $(j = 1, 2, 3)$ are constant of integration, which are obtained from the given boundary conditions.

Assuming an infinite series solution for u,

$$U = \sum_{n=0}^{\infty} Un \quad (18)$$

And the non-linear term by the infinite series

$$Nu = \sum_{n=0}^{\infty} An \quad (19)$$

Where *An* are the Adomian polynomial. Each terms of *An* can be uniquely computed by using the relation;

$$An = \frac{1}{n!}\left[\frac{d^2}{d\lambda^2} N\left(\sum_{n=0}^{\infty} \lambda^i U_i\right)\right]\lambda = 0, \quad i = 0, 1, 2, 3 \quad (20)$$

The *Un* are determined from the recursion algorithm

$$U_0 = \sum_{j=0}^{3} aj\frac{yj}{j!} + L^{-1}g, \ U_{n+1} = +\{-L^{-1}(RU + NU), n \geq 0\}$$

Where *Uo* is the zeroth component. The approximate solution of (15) is given as the truncated series

$$U(y) = \sum_{n=0}^{3} U_n \quad (14)$$

Where *U(y)* denotes the q-term approximation.

3.1 Adomian Decomposition Method of Solution

Applying ADM on equations (8) and (9), we have

$$L_1 f = S(\eta f^{111} + 3f^{11} - 2ff^{111}) + M^2 f^{11} \quad (23a)$$

$$L_2\theta = -PrS(2f\theta^1 - \eta\theta^1) + \text{Pr } Ec(f^{112} + 12\delta^2 f^{12}) \quad (23b)$$

Where

$$L_1 = \frac{d^4}{d\eta^4} \ and \ L_2 = \frac{d^2}{d\eta^2}$$

Applying $L_1 L_1^{-1}$ and $L_1 L_2^{-1}$ on both sides of (23a) and (23b) we have

$$f(\eta) = f(0) + \eta f^1(0) + \frac{n^2}{2!} f^{11}(0) + \frac{n^3}{3!} f^{111}(0) + L_1^{-1}\{S(\eta f^{111} + 3f^{11} - 2ff^{111}) + M^2 f^{11}\} \quad (24)$$

and

$$\theta(\eta) = \theta(0) + \eta_\eta \theta^1(0) + L_2^{-1}\{-\text{Pr } S(2f\theta^1 - \eta\theta^1) - \text{Pr } Ec(f^{111} + 12\delta^2 f^{12})\} \quad (25)$$

$$where \ L^{-1} = \int_0^\eta \int_0^\eta \int_0^\eta \int_0^\eta (.) \ d_\eta d_\eta d_\eta d_\eta \ and$$

$$L_2^{-1} = \int_0^\eta \int_0^\eta (.) \, d_\eta d_\eta.$$

From the associated conditions, we have

$$f(0) = A, \quad f^1(0) = 0, \ f(1) = \frac{1}{2}, f^1(1) = 0$$

$$\theta(0) = 1, \quad \theta(1) = 0 \quad (26)$$

Using equation (26) in (24) and (25) yields

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$$f(\eta) = A + \frac{n^2}{2!}A_1 + \frac{n^3}{3!}A_2 + L_1^{-1}\{S(\eta f^{111} + 3f^{11} - 2ff^{111}) + M^2 f^{11}\} \quad (27)$$

$$\theta(\eta) = 1 + \eta D + L_2^{-1}\{-\Pr S(2f\theta^1 - \eta\theta^1) - \Pr Ec(f^{112} + 12\delta^2 f^{12})\} \quad (28)$$

After substituting the Adomian polynomial, this produce, using maple

$$f_0(\eta) = A + \frac{\eta^2}{2}A_1 + \frac{\eta^3}{6}A_2$$

$$f_{n+1}(\eta) = L^{-1}[S(\eta f_n^{111} + 3f_n^{11} - 2f_n f_n^{111}) + L^{-1}(M^2 f_n^{11})] \quad (29)$$

$for\ n = 0$

$$f_1(\eta) = L[S(\eta f_0^{111} + 3f_0^{11} - 2f_0 f_0^{111}) + L^{-1}(M^2 f_0^{11})] \quad (30)$$

$$f_0 = A + \frac{1}{2}\eta^2 A_1 + \frac{1}{6}\eta^3 A_2$$

Hence,

$$f_1(\eta) = \left(\frac{-1}{12}SAA_2 + \frac{1}{8}SA_1 + \frac{1}{24}M^2A_1\right)\eta^4 + \left(\frac{1}{30}SA_2 + \frac{1}{120}M^2A_2\right)\eta^5 - \frac{1}{360}A_1SA_2\eta^6 - \frac{1}{2520}A_2^2S\eta^7 \quad (31)$$

Where A_n are the Adomian polynomials

$$\theta_0(\eta) = 1 + D\eta$$

$$\theta_{n+1}(\eta) = -L^{-1}[SPr(2f_n\theta_n^1 - \eta\theta_n^1) + PrEc\,(\textstyle\sum_{n=0}^{\infty} A_{n1} + 12\theta^2 \sum_{n=0}^{\infty} A_{n2})] \quad (32)$$

4.0 RESULT AND DICUSSION OF FINDINGS

Table 1: Comparison of Variational Iterative Method (VIM) solutions and Adomian Decomposition Method (ADM) solution for diverging channel for S=0.1, A=0.1, S=0.1, M=0.2. Pr = 0.3 and Ec = 0.2

.	$f^1(\eta)$		$\theta(\eta)$	
	VIM	**ADM**	**VIM**	**ADM**
0	0	0	1	1
0.2	0.384801	0.384801	0.806144	0.806144
0.4	0.575554	0.575554	0.607022	0.607022
0.6	0.575174	0.575174	0.407004	0.407004
0.8	0.384040	0.384040	0.206121	0.206121
1	0	0	0	0

Table 2: Values of skin friction coefficient $\frac{H^2}{r^2} Re_r\, Cf_r$ and the Nusselt number $(1-at)^{½}\, Nu$ for different values of A, S and M.

A	S	M	$\frac{H^2}{r^2} Re_r\, C_j$	$(1-at)$
-0.1	0.1	0.2	-3.62306	1.1317
0.0			-3.01553	1.0906
0.1			-2.40948	1.0568
0.2			-1.80490	1.0304
0.1	0.01		-2.40239	1.0581
	0.2		-2.41735	1.0553
	0.3		-2.42522	1.0583
	0.1	0.01	-2.40787	1.0568
		0.1	2.40828	1.0568
		0.2	2.40948	1.0568

Table 3: Values of Nusselt number $(1-at)^{½}\, Nu$ for different values of Pr, Ec and δ when A = 0.1, S=0.1, M = 0.2.

Pr	Ec	$(1\text{-at})^{½}$ Nu
0.0		1.00000
0.1		1.01894
0.2		1.03787
0.3	0.0	0.99860
	0.2	1.05680
	0.4	1.11501
	0.3	1.08487
		1.11074
		1.18836

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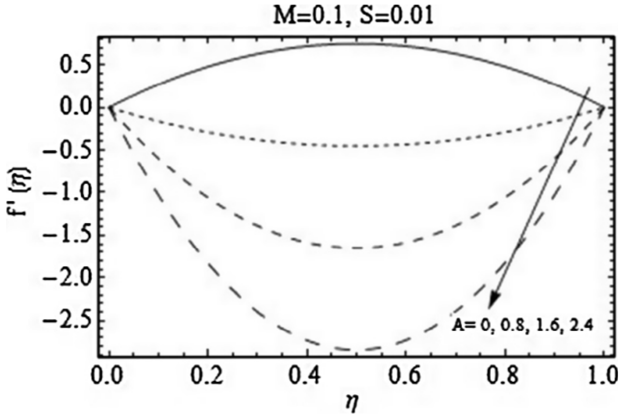
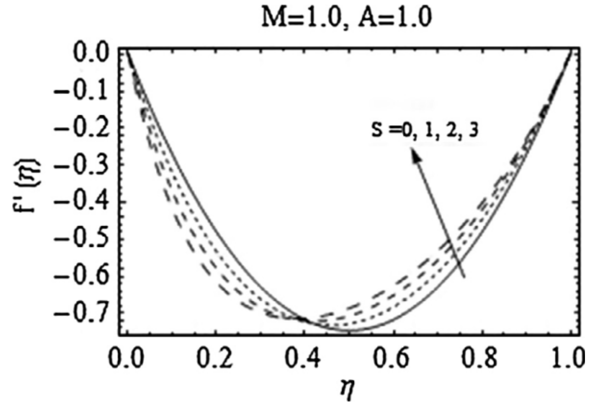
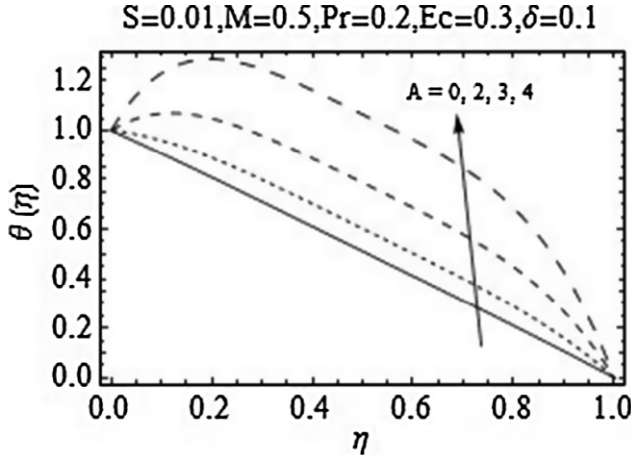
Graph (At suction case A >0)



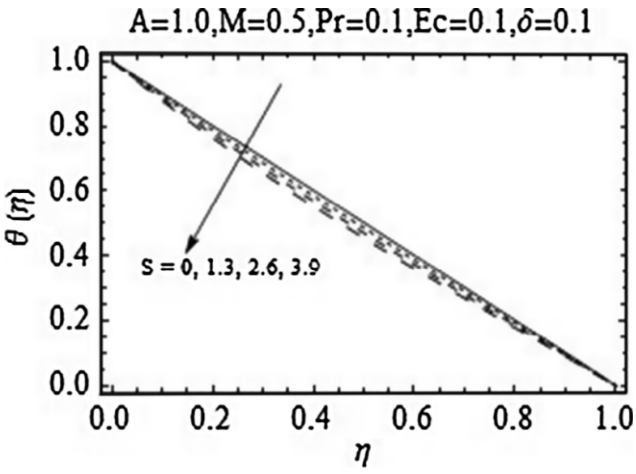
Effect of A on $f^1(\eta)$ Fig 1



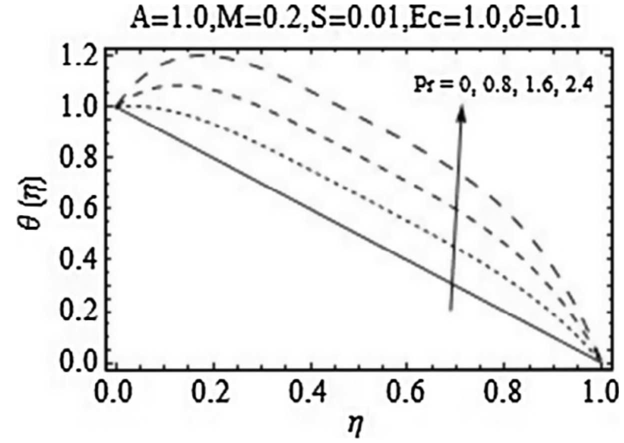
Effect of S on $f^1(\eta)$ Fig 1



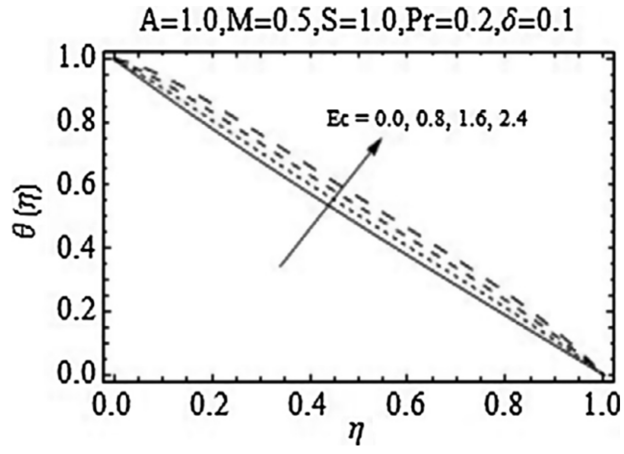
Effect of A on $\theta(\eta)$ Fig 3



Effect of S on $\theta(\eta)$ Fig 4



Effect of Pr on $\theta(\eta)$ Fig 5



Effect of Ec on $\theta(\eta)$ Fig 6

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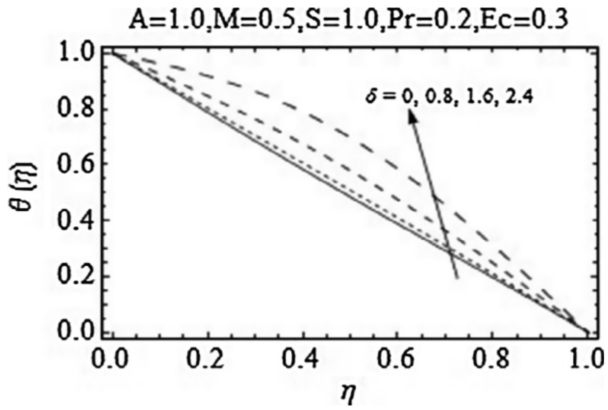
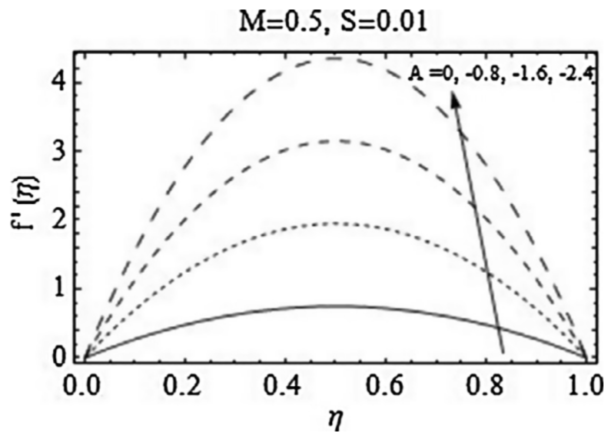
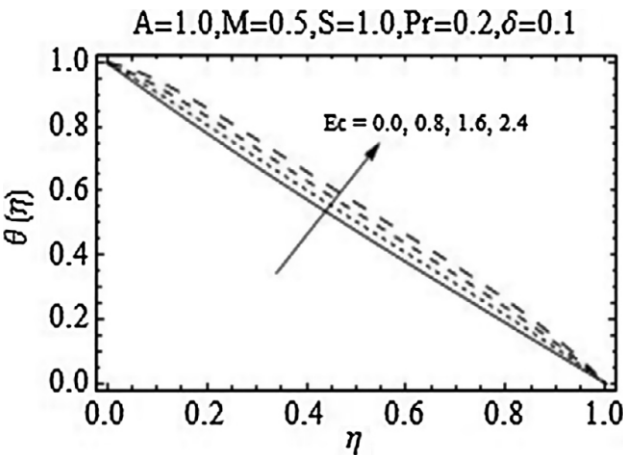


Effect of δ on $\theta(\eta)$ Fig 7

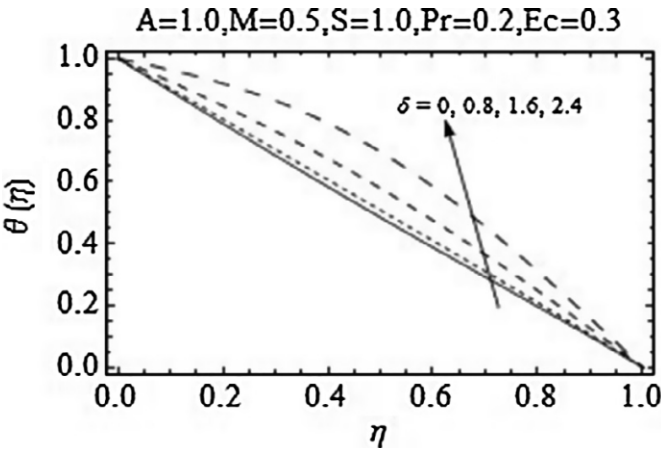
Graph (At Injection case $A < 0$)



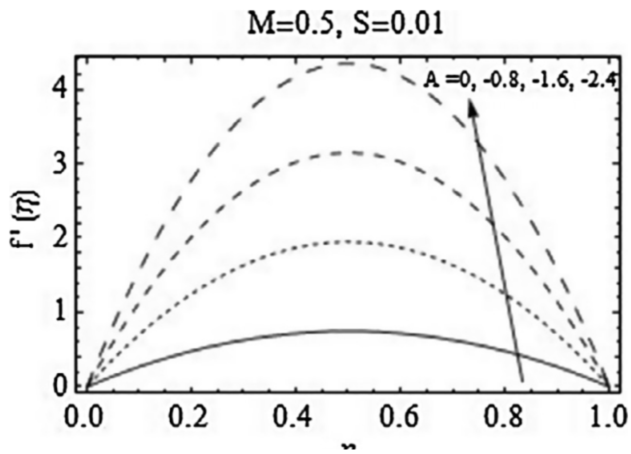
Effect of Pr on $\theta(\eta)$ Fig 5



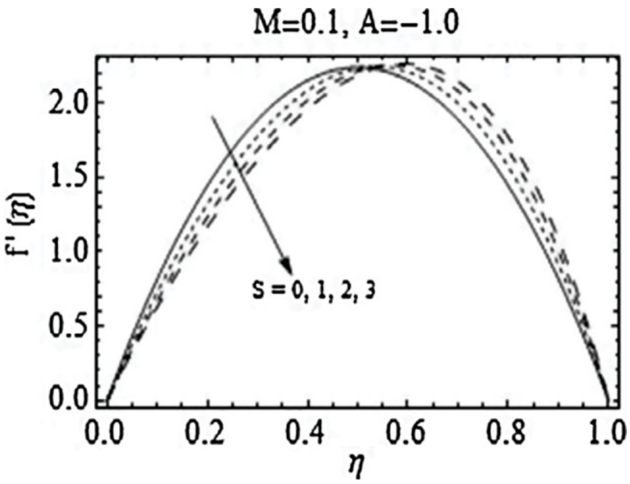
Effect of Ec on $\theta(\eta)$ Fig 6



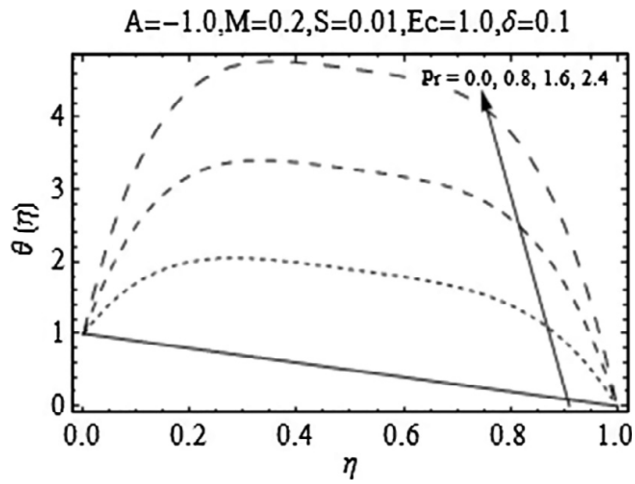
Effect of δ on $\theta(\eta)$ Fig 7



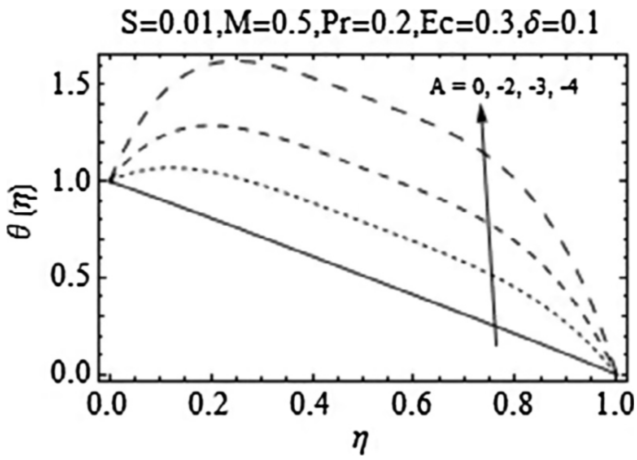
Effect of A on $f^1(\eta)$ Fig 8



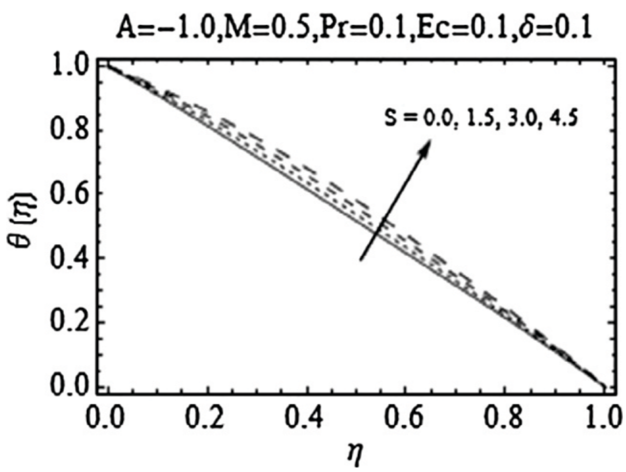
Effect of S on $f^1(\eta)$ Fig 9



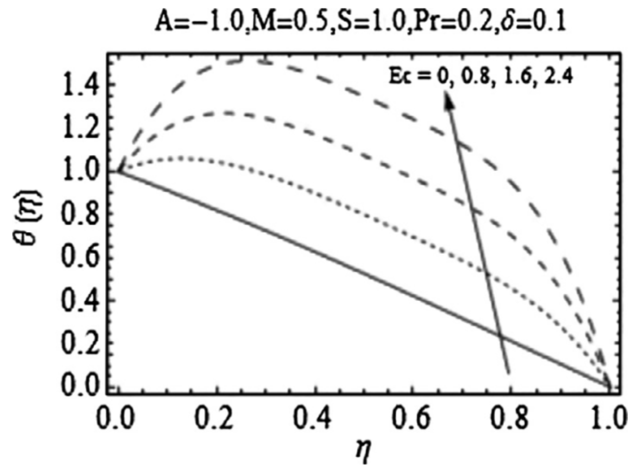
Effect of Pr on $\theta(\eta)$ Fig 10



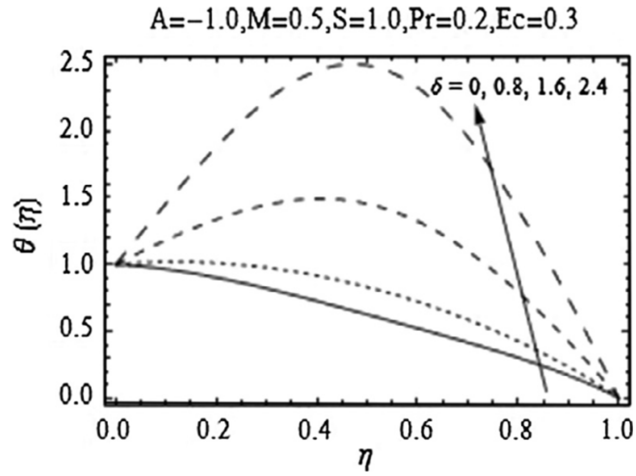
Effect of A on $\theta(\eta)$ Fig 11



Effect of S on $\theta(\eta)$ Fig 12



Effect of Ec on $\theta(\eta)$ Fig 13



Effect of δ on $\theta(\eta)$ Fig 14

Effect of different flow parameters on the velocity and temperature distributions are discussed in this section for both the suction and injection cases. Discussions are divided into two subsections; one is for suction case and the other for injection scenario.

4.1 Suction Case (A > 0)

Fig 1 shows the influence of porosity parameter A on the radial and axial velocity. It is clear from (1) that the axial velocity increases with increasing value of A. boundary layer thickness however is a decreasing function of A. due to the permeability of the upper disk when suction plays a dominant role it allows the fluid to flow near a the walls which results in a thinner boundary layer. Effects of deformation parameter S are displayed in fig 2.

S > 0 corresponds to the movement of upper disk away from the lower static disk while when S < 0 stands for its fall towards the lower disk. It can be seen from 2 that the absolute of $f^{I}(\eta)$ increases for increasing S in the range of $0 < \eta \leq 0.4$, while we have an opposite behaviour if absolute velocity is observed for $0.4 < \eta \leq 1$.

To exhibit the effect of flow parameters (in suction case) on the temperature profile (3-7) are presented. Fig 3 show $\theta(\eta)$ on increasing function of A, on the other hand thermal boundary layer becomes thinner with raising A. Consequences of increasing S are presented in Fig 4 according to which temperature profile falls with surging S.

Influence of Pr number on the temperature distribution is displayed in Fig 5 which declares $\theta(\eta)$ to be directly variant with Pr on the other hand thermal boundary layer is inversely proportional to Pr. it is due to the fact that for higher Pr low thermal conductivity is observed which results in narrow thermal boundary layer. Higher values of Prandtl number are associated with large viscosity oil, while lower Pr corresponds to low viscosity fluids. Fig 6 and 7 indicate that Eckert number Ecand dimensionless length $\theta(\eta)$ have similar effect on temperature profile as Pr.

4.2 Injection Case (A < 0)

Effect of physical parameters on velocity and temperature distributions for the case of injection are displayed in Fig 8 – 13. It is observed that on velocity profiles, effects of involved parameters are opposite to the ones discussed earlier for suction case. behaviour of temperature distribution however remain invariant for both the suction and injection cases.

Furthermore, table 1 shows a comparison between VIM solution and ADM solution. It is evident from the table that both the solution agree exceptionally well which shows the effectiveness of proposed analytical technique.

Table 2 and 3 display the numerical values of Nusselt number and coefficient of skin friction for different values of flow parameters. Absolute of skin friction is found to ben decreasing function of permeability parameter A. This observation is important due to its industrial implication since the amount of energy required to squeeze the disk can be reduced by increasing values of A.as discussed earlier, the suction parameter A decreases the thermal boundary layer thickness hence at the plates we have higher rate of heat transfer.

5.0 CONCLUSION

Heat transfer analysis is taken into account for the squeezing flow between parallel disks. ADM is used to determine the series solution to both velocity and temperature distributions. Main upshots of this study are presented a below:

- It is observed that the value of ADM and VIM are accurate.
- The behaviour of all physical parameters is opposite on velocity profile in the cases of suction and injection. On the other hand, the effect of parameters remain similar on temperature profile for both suction and injection cases.
- Temperature $\theta(\eta)$ is directly proportional to the Prandtl number.

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